

Network Scale-up Methods on Aggregated Relational Data to Estimate the Outcome of Elections

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Abstract

The Network Scale-Up Method (NSUM) is an estimation framework that aims to determine the size of hidden or hard-to-reach populations from questions such as “How many people do you know who belong to the target population?” The information collected from these questions is commonly referred to as aggregated relational data (ARD), and the estimation of hidden population sizes using NSUM in ARD has been widely applied to key problems in sociology and public health. Note that this approach has been widely used to estimate the size of populations subject to legal or social restrictions, such as sex workers and drug users, who are typically excluded from the formal census. Although voting intention is not a social stigma, this information has become a privacy-sensitive issue, particularly in polarized political contexts, and thus poses a challenge to determining the share of party support. In this work, we introduce a methodology for estimating vote-intention shares using NSUM techniques on ARD. The methodology involves the design of the indirect survey to collect ARD regarding the voting intention of the survey participant’s contacts, jointly with other control questions, the processing of the data with appropriate filters to eliminate outliers, the study of sample stratification strategies, and finally, the support share estimation for each political group by using different NSUM techniques. The methodology is applied to estimate voting outcomes in the 2023 Spanish general elections, using the Madrid, Andalusia, and Valencia regions as experimental scenarios. Overall, the resulting estimates **are competitive with** those published by leading private and public survey institutes, despite using a significantly smaller number of participants.

Keywords: Aggregated Relational Data, Indirect surveys, Network Scale-Up Methods, Voting Intention Shares.

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1 Introduction

Direct surveys have been commonly used to estimate election results. In recent years, the results of surveys conducted by companies and institutions have not been free of public controversy due to their poor estimates. Many factors could explain this finding, such as coverage errors due to underestimating segments of the population, a poorly designed sampling strategy, or biases introduced by undecided voters in surveys, among others. Recent examples include surveys for the July 2023 general elections in Spain, which failed to estimate parliamentary seats [FRB24], and surveys for the first round of the October 2023 presidential elections in Argentina, which failed to predict the leading candidate.

An alternative to direct surveys is indirect reporting, in which respondents are asked to report the number of their contacts in specific groups. This approach has at least three significant advantages over direct surveys. First, the surveys are designed to avoid the collection of personal information from participants and thus protect their privacy (i.e., they make the study anonymous), thereby [reducing some privacy risk](#). Secondly, they have broader coverage with the same number of participants, since respondents are asked about their contacts. Thirdly, there are techniques that allow estimation of the

subpopulations of interest in the survey, known in the literature as network scale-up methods (NSUM), which, in a broad sense, refer to the process of estimating network or subpopulation size using aggregated relational data (ARD) [LBN21]. ARDs are obtained by reporting on the different groups known to the respondent (e.g., their network of contacts).

In general, direct surveys have limitations when dealing with the study populations sharing some “sensitive” characteristic, like, for example, sex workers, LGBTQ+ individuals, people with HIV/AIDS, refugees and immigrants, formerly incarcerated individuals, religious or ethnic minorities, etc., due to social stigma. This drawback also applies to electoral surveys, as many individuals may conceal their political opinions for privacy reasons.

It is important to note that over the past 35 years, research on NSUM has focused on quantifying the size of such populations from ARD. Nevertheless, these techniques have not been widely applied in public opinion surveys used to estimate the electoral support for political parties, making our proposal an innovative contribution to this field. Several previous works have used indirect surveys to estimate electoral results [Mur11] [GBdBD⁺18] [Gra14] [Gra16] [BdBGB⁺22]. However, none of them present how to use the NSUM methods to estimate electoral outcomes, so they have not analyzed the behavior of existing methods (stochastic, frequency-based, etc.). Additionally, none of these works described how to prepare the data collected from indirect electoral surveys to make the estimations, nor do they present a methodology for using indirect surveys in electoral processes.

This article proposes a methodology based on indirect surveys and NSUM to estimate electoral process voting outcomes. In that sense, our methodology proposes a framework for conducting indirect surveys and then applies NSUM-based approaches to estimate vote percentages by political group. Furthermore, the article examines the indirect reporting approach in the context of the July 2023 Spanish general elections. Thus, the main contribution is the design of a methodology based on indirect surveys and the NSUM to estimate electoral outcomes. Some other contributions of this work are:

- The design of a survey structure/format of indirect surveys for electoral contexts.
- The characterization of filters to clean surveys of atypical values.
- The comparison of different NSUM approaches in estimating election outcomes. In particular, the following methods were compared: the Maximum Likelihood Estimator (MLE), the Plug-in Maximum Likelihood Estimator (PIMLE), the Means of Sums (MoS), and the Naïve.
- A comparative analysis of the results of our approach with those of other direct surveys conducted in Madrid, Andalusia, and Valencia during the aforementioned electoral process.

The rest of the paper is organized as follows: the next section deals with an overall review of related work about NSUM. Sections 3 and 4 are the core parts of our work as they present a unified methodology covering the survey design and data collection, the data processing step, and the application of NSUM approaches for the estimation of election outcomes in real cases. Specifically, Section 3.3 addresses the approaches based on NSUM that will be applied to the estimation of elections voting outcomes. The methodology is applied to the Spanish general elections of July 23rd, 2023 in Madrid, Valencia, and Andalusia regions. Section 5 describes the resulting estimations, which are compared with the results published by private and public survey houses and with the ground truth outcome. [Finally, Sections 6 and 7 finish this work with a discussion of the application of the proposed methodology and some concluding remarks.](#)

2 Related Work

Indirect surveys have [emerged](#) as a strategy to collect information quickly and anonymously, and then estimate the size of hidden subpopulations from such indirect survey data. To estimate unknown subpopulations, it is necessary to collect information about the contact network of surveyed individuals. The initial idea behind the NSUM was introduced by Bernard et al. [JKR89], and consists of using ARD to obtain the network size (degree) of the participants, and then uses this information to estimate the unknown subpopulation. Subsequently, works based on this approach continue to appear [KJM⁺98a, KMB⁺98b, HDK15a]. Particularly relevant is the estimator by Killworth et al. [KMB⁺98b] since it has become the most employed NSUM estimation procedure [EMN⁺12, SSB⁺12, WYZ⁺15, NSS⁺16]. The NSUM usually uses questions about well-known groups to estimate each surveyed individual network size. Also, McCarty et al. [MKB⁺01] use several questions that cover all the possible acquaintances, and then assign the degree

to the sum of the responses. Additionally, García-Agundez et al. [GAOHR⁺21] directly ask about the size of the personal network. The NSUM approaches have been applied in epidemiological problems [UNA17, MRMB15, TPC⁺19, FUM⁺16], health and social issues such as drug users [KKBB06, SFM⁺10], cancer patients [CSMG17], abortions [SGAD20], and sex workers [JLC⁺18].

On the other hand, several works have been developed to address the estimation procedure with NSUM. New Bayesian models have been proposed to handle biases [ZSG06, MZ12, MRMB15, TPC⁺19] and the correlation between groups [LBN23]. Other works add direct survey data in order to create new estimators or adjust previous methods [FS16, VWR⁺19, PNB22]. Finally, ARD has been used as well to produce a latent network representation [MZ15], reconstruct network [BCMP20], and estimate network properties [BCMP19, BCL⁺23]. Two review works about NSUM and ARD [LBN21, McC20] collect most of the existing approaches in the literature.

In the context of electoral surveys, De Sio et al. [SW20] define a model that explains election outcomes on the basis of party strategy. The article traces a three-step process: (1) The configuration of electoral risks and opportunities (which is captured through public opinion surveys) guides party communication (measured with Twitter data) such that (2) parties design their campaigns strategically, which in turn, (3) improves their electoral performance (measured using official statistics). The article [SSBT20] analyses the second-order election (SOE) model originally developed by Reif and Schmitt in 1980, which describes voter behaviour in the second round of two-round elections. This model suggests that, relative to the behaviour described by the first-order model (describing voter behaviour in the first round), turnout is lower, large parties lose and small and ideologically extreme parties win. The article [SSBT20] analyses such hypotheses of the SOE model in the 2004 and 2014 European elections. Skoris et al. [SLJ20] report a meta-analysis examining the predictive power of social media data by focusing on (1) sentiment analysis and (2) analysis of structural features. The study shows that machine learning-based estimates are generally better than those derived from pre-existing lexica and that a combination of structural features and sentiment analyses provides the most accurate predictions.

Some other works have exploited direct surveys to analyze specific aspects of electoral processes: polarization, the behavior of sectors of the population, opinions on specific issues, etc., which is an interesting preamble to the use of indirect surveys for these same studies. For example, Hernandez et al. [HAR21] analyze the effects of the election on affective polarization. The article studies the origins of affective polarization by exploring two mechanisms: changes in ideological polarization and in the intensity of party identification. De Geus et al. [dGRMS22] investigate the prevalence and correlates of sexism in the British political context, using a measure of ambivalent sexism that distinguishes between hostile and benevolent sexist attitudes. They use two online surveys and find that more than half of the population holds some sexist attitudes which can be predicted by gender, education, religiosity, and authoritarian values. The work of Schroedel et al. [SBDR20] analyzes the impact of political trust on Native American electoral participation using direct surveys. They use in-person survey responses conducted in Native communities with the support of tribal leaders. Respondents expressed high levels of distrust in nontribal government and voting methods, and this distrust had an impact on the decision to participate in nontribal elections. As we can see, these latest works touch on sensitive issues such as polarization, sexism, and trust in government. Indirect surveys could improve the quality of the information retrieved by expanding the coverage and by protecting the privacy of participants (they may respond more truthfully and thus produce less bias).

There are other works that have discussed the use of indirect surveys and, especially, of collective questions that inquire about a population or the immediate social circles of respondents, in a variety of elections in different countries. For example, Murr [Mur11] explored the “wisdom of crowds” in forecasting elections. The approach relies on citizen prediction, which aggregates citizens’ local expectations to directly forecast constituencies. Using the 2010 British Election Study data, the study highlighted that group size significantly influences the accuracy of predictions. This approach can directly forecast seat shares, although sometimes it yields a high 15% error. Another aspect, which is not analyzed by the approach, is the number of respondents needed to ensure a good forecasting capacity. Finally, the aggregation of local expectations to predict does not consider the effect of electoral campaigns, which can influence the opinion of the respondent. On the other hand, Galesic et al. [GBdBD⁺18] demonstrated that social-circle questions, which inquire about the voting intentions of participants’ social contacts, offer improved predictions. These questions accurately forecasted the 2016 US and 2017 French presidential election outcomes on national, state, and individual levels. They also helped explain last-minute voting intention changes and provided insights into the dynamics of echo chambers among supporters of different candidates. The article describes how the survey was conducted (waves of study) that differed in the types of questions asked (three types of questions: questions about one’s own intention,

about one’s social circle and about the winner of the election) and when they were asked (they were asked together, separately, etc.), but it does not give details of how the survey results were estimated (nor does it show these results). Graefe [Gra14] compared the accuracy of vote expectation surveys for forecasting US presidential elections from 1932 to 2012 to predictions from four established methods: expert judgment, traditional polls, prediction markets, and quantitative models. Also, Graefe [Gra16] evaluated vote expectation surveys for forecasting elections in the 2013 German election. It concluded that representative surveys are a valuable, low-cost method for forecasting, especially for small-scale elections where traditional methods like representative polls or prediction models may not be feasible or cost-effective. One concern with using this approach is that people’s expectations may be influenced by their preferences. In the case of elections, this means that respondents tend to predict that their preferred candidate will win. While this approach does follow Condorcet’s jury theorem, which implies that the probability of a group reaching a correct decision increases rapidly toward 100 percent as the number of individual estimates increases, this bias is not eliminated. In other words, even if you use Condorcet’s jury theorem which combines individual estimates into one group estimate, this is correct if you manage to distribute the sample well across all candidates. Finally, a recent work of Bruine de Bruin et al. [BdBGB⁺22] found that social-circle questions outperform own-intention questions in predicting elections with numerous political parties, specifically the Netherlands’ 2017 general election and Sweden’s 2018 general election. This suggests that social-circle questions remain effective even in complex electoral environments. However, the work does not detail the characteristics of the samples, the environment, or how the calculations are carried out to ensure good results.

In the reviewed literature, we find methods based on indirect reports according to the concept of social-circle questions, but they do not provide a unified methodological framework, and do not show how to use the NSUM methods to estimate the result of an electoral process by assessing the voting intention for each political group. This is relevant since there are different NSUM approaches (frequentist, Bayesian, and network models), each one having properties that advise its suitability for the application in specific cases. Knowing them and how to use them, would allow us to know the method that better fits each context. Furthermore, there are no relevant works that indicate how to prepare the data collected from electoral indirect surveys, and in general, there is a research gap in the literature dedicated to analyzing the quality of electoral indirect surveys.

3 Methodology

Our proposal relies on a methodology that unifies the main steps of the estimation problem as follows: First, data are collected via indirect surveys, for which a well-suited design is required in advance. In the second step, the data are processed to improve data quality by removing irrelevant information (e.g., outliers or aberrant responses). Finally, standard NSUM procedures (see Table 1) are used to estimate the election results for each political group. Hence, the result of the methodology will be the estimations of the vote proportion for each political party: \hat{p}_u^{Naive} , \hat{p}_u^{MoS} , \hat{p}_u^{MLE} and \hat{p}_u^{PIMLE} . Each step of the proposed methodology is described in the next sections.

3.1 Survey Design and Data Collection

At this stage, the basic requirements for implementing the NSUM are established. They are concerned with the following issues involved in survey design and data collection:

- Selection of the digital survey platform used to carry out the social network survey. Pollfish or Prolific are examples of well-established platforms in the market.
- The sampling framework which should include at least the geographic and socio-demographic limits of the survey.
- The size of the survey, which is usually limited by budget restrictions.
- The preparation of the questionnaire that includes the “how many...” questions about the political options and blank vote (it includes blank ballot, abstention, and “other parties”), as well as questions regarding the auxiliary known populations.
- The possible stratified sampling strategy by using the age, gender, or some other social attributes of surveyed individuals.

In particular, we used the Pollfish platform to conduct online surveys, which is age-biased (unbalanced toward younger respondents). The first time we used it for the regional elections in Madrid (see Supplementary Material A.3), we observed this imbalance, so we proposed a post-stratification strategy (i.e., the definition of population weights). In the case of stratified sampling, it was implemented directly on the platform, and age was selected as the stratification variable because the sample’s age distribution differed from the population distribution. The ideal is to conduct a study of all potential stratification variables to select the one to use. Thus, several strategies could be proposed: (i) choose the variable whose stratified sample is closest to the real one; (ii) carry out sampling using a different variable in each round; (iii) use a metric from information theory (for example, entropy) to select the variable that provides more information. This is one possible extension of this work.

3.2 Data Processing

Once the survey has been conducted, the collected responses must be processed. This phase concerns the preparation of the survey data. A key task is data cleaning to eliminate inconsistent answers and outliers. To do this, a set of rules must be established in advance to ensure data quality in accordance with specified criteria. Such criteria should include at least the following points:

- Identification and elimination of cases with inconsistent (abnormal) network size.
- Identification and removal of responses declaring an abnormal number of blank voting.
- Identification and elimination of too much polarized responses.

Their specific application to real use cases is addressed in Section 4.2.

3.3 Estimation of Voting Intention

We propose to use NSUM for estimating the size of populations sharing some “sensitive” characteristic, which in our case **corresponds** to the number of potential voters for each political group. Our proposal is based on the simple idea of assessing these populations through indirect survey data, which collects the voting intention for the different political parties that participate in the electoral process, by means of questions about the network of respondents as follows: “How many people do you know who will vote for political party X?”. Our approach mitigates many legal privacy concerns, as respondents provide information only about non-identifiable individuals in their networks.

In this context, we must consider different sources of uncertainty arising from indirect survey reporting. The main ones can be described as follows: (i) *barrier effects* are concerned with biases produced by the fact that the probability of knowing someone in a population depends on the characteristics of the participants; (ii) *transmission errors* are produced by the fact that the participant does not know the status of their acquaintances; (iii) *recall errors* occur when the participants inaccurately remember the number of their contacts that belong to some subpopulations. NSUM approaches are influenced by the previous sources of uncertainty.

The NSUM relies on the following simple idea: first, the degree (network size) of respondents is estimated using a set of auxiliary questions about the ego network for populations of known sizes (usually data from official statistics). Then, the survey data collected from participants’ networks are combined with the degree estimates to construct an estimate of the unknown target population. In order to formalize the previous ideas along with their implementation by different NSUM estimation strategies, we now introduce some notation:

- Let N be the size of the overall population.
- Let n be the size of the survey (number of respondents).
- Let Q be the number of available political options and let us denote by N_u the unknown size of the population voting for the u -th political group (blank vote option included): $u = 1, \dots, Q$.
- Let y_{iu} be the number of people the respondent i declares to know that will vote for a specific political option u , and let us denote by $y_i = (y_{i1}, \dots, y_{iQ})$ the vector collecting such responses for all the political groups.
- Let L be the number of auxiliary subpopulations, with known sizes $\mathcal{N}_k : k = 1, \dots, L$, which are usually obtained from official statistics.

Method	Degree estimation	Estimation of hidden population	Reference
PIMLE	$\hat{d}_i = N \cdot \frac{\sum_{k=1}^L x_{ik}}{\sum_{k=1}^L \mathcal{N}_k}$	$\hat{N}_u^{PIMLE} = \frac{1}{n} \sum_{i=1}^n N \cdot \frac{y_{iu}}{\hat{d}_i}$	[KJM ⁺ 98b]
MLE	$\hat{d}_i = N \cdot \frac{\sum_{k=1}^L x_{ik}}{\sum_{k=1}^L \mathcal{N}_k}$	$\hat{N}_u^{MLE} = N \cdot \frac{\sum_{i=1}^n y_{iu}}{\sum_{i=1}^n \hat{d}_i}$	[KMB ⁺ 98a]
MoS	$\hat{d}_i = \frac{1}{L} \sum_{k=1}^L N \cdot \frac{x_{ik}}{\mathcal{N}_k}$	$\hat{N}_u^{MoS} = \frac{1}{n} \sum_{i=1}^n N \cdot \frac{y_{iu}}{\hat{d}_i}$	[HDK15b]
MoS2	$\hat{d}_i = \frac{1}{L} \sum_{k=1}^L N \cdot \frac{x_{ik}}{\mathcal{N}_k}$	$\hat{N}_u^{MoS2} = N \cdot \frac{\sum_{i=1}^n y_{iu}}{\sum_{i=1}^n \hat{d}_i}$	—
Naïve	$\hat{d}_i = \sum_{v=1}^Q y_{iv}$	$\hat{N}_u^{Naive} = \frac{1}{n} \sum_{i=1}^n N \cdot \frac{y_{iu}}{\hat{d}_i}$	—
Naïve2	$\hat{d}_i = \sum_{v=1}^Q y_{iv}$	$\hat{N}_u^{Naive2} = N \cdot \frac{\sum_{i=1}^n y_{iu}}{\sum_{i=1}^n \hat{d}_i}$	—

Table 1: Standard NSUM approaches based on sums and averages.

- Let us denote by x_{ik} the answer of respondent i to the k -th auxiliary question, and let us denote by $x_i = (x_{i1}, \dots, x_{iL})$ the responses given by respondent i to the L auxiliary questions.

The first ad hoc strategy would ignore the survey data from the auxiliary populations and would use the network vote declaration for the different political groups to estimate the network size of respondent i by $\hat{d}_i = \sum_{v=1}^Q y_{iv}$. Hence, the Naïve estimator for the unknown N_u would be

$$\hat{N}_u^{Naive} = \frac{1}{n} \sum_{i=1}^n N \cdot \frac{y_{iu}}{\sum_{v=1}^Q y_{iv}} : u = 1, \dots, Q. \quad (1)$$

A second Naïve strategy to estimate N_u would rely on the extrapolation to the general population of the declared voting intention to political group u by all the respondents to the survey. Such extrapolation would be given by

$$\hat{N}_u^{Naive2} = N \cdot \frac{\sum_{i=1}^n y_{iu}}{\sum_{v=1}^Q \sum_{i=1}^n y_{iv}} : u = 1, \dots, Q. \quad (2)$$

Standard NSUM approaches use sums and averages as the building blocks to obtain the estimation of an unknown target population. The most popular methods include the Maximum Likelihood Estimator (MLE), the Plug-in Maximum Likelihood estimator (PIMLE), and the means of sums (MoS). The mathematical expressions giving the estimations of the degree and the size of the unknown population are shown in Table 1. The table uses the notation employed in the review work [LBN21] and in the references to the seminal works that proposed each method. While MLE relies on random mixing assumptions—specifically, that the reported number of contacts in the hidden population follows a Binomial distribution with probability of success equal to the proportion of the hidden population—PIMLE and MoS only assume that the ratios y_{iu}/\hat{d}_i are representative on average. All the referenced approaches are implemented in the `networkscaleup` R package [LBN22].

Note that a second alternative for the MoS method would arise from the following combination (see the fourth row of Table 1).

$$\hat{N}_u^{MoS2} = N \cdot \frac{\sum_{i=1}^n y_{iu}}{\sum_{i=1}^n \hat{d}_i} \text{ with } \hat{d}_i = \frac{1}{L} \sum_{k=1}^L N \cdot \frac{x_{ik}}{\mathcal{N}_k}. \quad (3)$$

For the sake of completeness, we are also including in the table the expressions for the classical estimators, along with expressions for both Naïve estimation methods given by formulas (1) and (2), and the MoS2 method (3).

Let us denote by \hat{N}_u the estimator of the number of votes obtained by a scale-up method for any of the available political choices: $u = 1, \dots, Q$. Once the size, N_u , of the unknown population has been

estimated, we propose to estimate the voting percentage of the u -th political group by means of the following expression:

$$\hat{p}_u = \frac{\hat{N}_u}{\sum_{v=1}^Q \hat{N}_v} : u = 1, 2, \dots, Q. \quad (4)$$

Depending on the network scale-up method being applied, we will get the versions Naïve, Naïve2, MoS, MoS2, MLE and PIMLE for the estimation of the proportion. The estimations obtained by these methods will be denoted as follows: \hat{p}_u^{Naive} , \hat{p}_u^{Naive2} , \hat{p}_u^{MoS} , \hat{p}_u^{MoS2} , \hat{p}_u^{MLE} and \hat{p}_u^{PIMLE} , where the superscript indicates the method and the subscript denotes the political group.

It is worthwhile noting that:

$$\hat{N}_u^{Naive2} = N \cdot \hat{p}_u^{Naive2} \text{ and } \hat{N}_u^{MoS2} = N \cdot \hat{p}_u^{MoS2}$$

since $\sum_{v=1}^Q \hat{N}_v^{Naive2} = \sum_{v=1}^Q \hat{N}_v^{MoS2} = N$.

In this work, we only consider the versions provided by the methods MLE, PIMLE, MoS, and Naïve because it can be proved that $\hat{p}_u^{MLE} = \hat{p}_u^{MoS2} = \hat{p}_u^{Naive2}$, as the next proposition shows.

Proposition 1. *Let us consider the estimators \hat{p}_u^{MoS2} , \hat{p}_u^{Naive2} , and \hat{p}_u^{MLE} of the voting proportion, being defined by the quantities:*

$$\hat{p}_u^{Naive2} = \frac{\hat{N}_u^{Naive2}}{\sum_{v=1}^Q \hat{N}_v^{Naive2}}, \hat{p}_u^{MoS2} = \frac{\hat{N}_u^{MoS2}}{\sum_{v=1}^Q \hat{N}_v^{MoS2}} \text{ and } \hat{p}_u^{MLE} = \frac{\hat{N}_u^{MLE}}{\sum_{v=1}^Q \hat{N}_v^{MLE}},$$

respectively. Then, it holds that

$$\hat{p}_u^{Naive2} = \hat{p}_u^{MoS2} = \hat{p}_u^{MLE}.$$

Proof. From the expressions of the MLE estimator that appear in Table 1, it can be obtained that:

$$\hat{p}_u^{MLE} = \frac{\sum_{i=1}^n y_{iu}}{\sum_{v=1}^Q \sum_{i=1}^n y_{iv}}.$$

On the other hand, as the sum $\sum_{i=1}^n \hat{d}_i$, with \hat{d}_i being the estimator of the degree in \hat{N}_u^{MoS2} , does not depend on the unknown voting populations, we get:

$$\hat{p}_u^{MoS2} = \frac{\sum_{i=1}^n y_{iu}}{\sum_{v=1}^Q \sum_{i=1}^n y_{iv}} = \hat{p}_u^{MLE}.$$

Using similar arguments, we also obtain that $\hat{p}_u^{Naive2} = \hat{p}_u^{MLE}$. \square

It can also be noted that when the PIMLE method is applied to expression 4, it turns out a quantity independent of the totals \mathcal{N}_k in the auxiliary information. This fact is proved by simple calculations as follows: from the first row of Table 1 we get

$$\hat{N}_u^{PIMLE} = \frac{1}{n} \sum_{i=1}^n \frac{y_{iu}}{\frac{\sum_{k=1}^L x_{ik}}{\sum_{k=1}^L \mathcal{N}_k}} = \left(\frac{1}{n} \sum_{k=1}^L \mathcal{N}_k \right) \sum_{i=1}^n \frac{y_{iu}}{\sum_{k=1}^L x_{ik}},$$

which in turn implies that

$$\hat{p}_u^{PIMLE} = \frac{\hat{N}_u^{PIMLE}}{\sum_{v=1}^Q \hat{N}_v^{PIMLE}} = \frac{\sum_{i=1}^n \frac{y_{iu}}{\sum_{k=1}^L x_{ik}}}{\sum_{v=1}^Q \sum_{i=1}^n \frac{y_{iv}}{\sum_{k=1}^L x_{ik}}}$$

as we have just asserted.

4 Application of the Methodology

The proposed methodology is applied to the Spanish general elections of July 23rd, 2023 (national elections) in specific communities (Madrid, Andalusia, and Valencia). The application of each step of the methodology is described in the next sections.

4.1 Survey Design and Data Collection

The data were collected by indirect survey reporting using the facilities provided by Pollfish Survey Tools (<https://www.pollfish.com/>). Below, we describe the survey design and how the information has been obtained.

- One round was conducted during the electoral campaign (fifteen days before the voting day).
- We do not limit the number of contacts of surveyed individuals so it is left open for him/her to specify the size of his/her network.
- **Sampling framework:** The surveyed people are individuals who live in the Madrid, Andalusia, or Valencia regions, are at least 18 years old, and have a potential voting network in these regions. Moreover, the respondents are asked only about their social network contacts who belong to these regional electoral censuses.
- **Sample size:** $n = 200$ with stratification by age for the general elections (more details about stratification in Section 4.3).
- **Target unknown populations:** The main political groups presenting candidates for the general elections: PP, PSOE, Vox, and Sumar, the latter arising from the alliance between Más Madrid and Podemos. Adding the option of Blank vote yields in this case $Q = 5$.
- **Auxiliary known populations:** People with age in the intervals $(18, 30]$, $(31, 54]$ and > 54 , self-employed people, unemployed, and physicians who belong to the respondent network. The real data for the auxiliary populations were obtained from official statistics provided by the Instituto Nacional de Estadística (INE, <https://www.ine.es/>), a public institution that manages all official statistics in Spain.
- **Questionnaire:** Both data sources, target and auxiliary, are collected from respondents by asking “How many people who ... do you know?” questions.

Additionally, the Pollfish platform provides demographic information on surveyed individuals, including age, income, and education. Note that the survey design assumes respondents are already aware of their contacts’ voting intentions; that is, the questionnaire did not prompt respondents to inquire about these intentions directly during data collection. To mitigate potential biases arising from uncertainty about contacts’ voting intentions, the survey includes control questions that assess respondents’ familiarity with the demographic characteristics of their immediate environment, e.g., “How many of your contacts considered above are between 18 and 34 years old?” (see Section A.1 in the Supplementary Material). Note also that MLE and PIMLE methods indirectly use these control questions to determine voting intention. Specifically, these methods integrate the control questions to determine the size of each respondent’s network [KJM⁺98b, KMB⁺98a].

The estimand in our study is the proportion of voting intentions among contacts for whom the respondent knows the political preference. That is, we estimate vote shares within the subnetwork of politically legible ties, rather than the full social network. In this study, respondents’ political preferences are unavailable. We observe aggregated political preferences of their contacts, as reported by the respondents. This distinction is important for understanding what is being measured and how the results should be interpreted. The analysis relies on several arguments that explain why the observed preferences of contacts can still provide a useful approximation of broader political patterns. First, the analysis is conducted in a high-salience context. During general elections, political discussion is widespread, and voting intentions are often disclosed in social contexts, such as conversations with family, friends, or coworkers. This increases the likelihood that respondents are aware of the political preferences of a meaningful portion of their contacts. Second, the NSUM estimators use aggregate information from many respondents. Hence, individual-level selection effects, such as differences in political knowledge among respondents, partially average out when computing overall estimates. Taken together, these elements support the idea that, even without direct information on respondent preferences, the reported preferences of contacts can offer a reasonable and informative view of political support patterns in the population.

Finally, barrier effects in NSUM are not solely determined by age but also by the interaction between respondent characteristics and the social visibility of each target group. Different political groups may face distinct barriers depending on polarization, stigma, group size, and the contexts in which political discussions occur. In our analysis, age stratification and known population corrections mitigate some

of these effects but do not eliminate all sources of heterogeneity. For this reason, we interpret the estimates as aggregates that average across respondent- and group-specific barriers. We complement these considerations with a sensitivity analysis in the Supplementary Material (Section C), which examines the performance of the various NSUM approaches under deviations from random mixing.

4.2 Data Processing

Data cleaning is needed to remove inconsistent responses from people who claim to know an aberrant number of voters for a specific political party, or who report an abnormal number of blanks in their network. There are also respondents who report knowing the voting intention of a highly unusual number of contacts in their social network; such cases are more likely to result in *recall errors*, so we should account for them by implementing specific filtering to remove them before computing the NSUM estimation. Based on these considerations, we establish several rules for assessing and removing outliers and “dirty” information. The following data cleaning criteria are considered in advance:

- Removal of the responses at the top 20% of voting declaration network size $\sum_{u=1}^Q y_{iu}$. This criterion results from an exploratory data analysis using a boxplot, a statistical visualization method that allows for outlier identification. Successive applications of the boxplot are performed at different atypical thresholds until we obtain a boxplot with no outliers. In all the cases under study, the resulting boxplots suggest the elimination of the top 20% network sizes.
- Removal of the responses declaring a network with blank voters above a specific threshold. The median absolute deviation (MAD) criterion, with a deviation of 5, is used to set the blank voting threshold.
- Removal of the responses declaring a majority (above 95%) of the network voting intention to a specific party.

Based on the previous rules, several scenarios are possible: the first one would be not applying any filtering criteria and processing all the survey responses. The second one would involve either using these rules separately or combining them to filter out surveys that may contain inconsistent responses. For the sake of completeness, the appendix with the Supplementary Material also includes tables (see Table SM1) displaying the estimations that result from the application of NSUM procedures when no filtering is considered. Just to illustrate the number of surveys removed after data cleaning, we observe that the simultaneous application of the previous rules would result in 128 surveys.

On the other hand, a simple exploratory data analysis warns of potential estimation errors arising from *barrier effects* (see [LBN21]). An illustrative example is shown in Figure 1, which shows contingency tables for the distribution of the age and the voting declaration of the network against the age of the respondents for a survey without any stratification deployed in [Madrid regional election process of May 2023 to estimate the voting intention](#). Here, we provide only contingency tables for the indirect survey data collected in one of the rounds deployed (see the Supplementary Material for more information about this survey).

4.3 Sampling Stratification for NSUM-based Estimation Procedures

In order to handle the biases that may arise from survey data collection, we propose one alternative for adapting NSUM procedures to the vote estimation problem. This alternative relies on the simple idea of introducing stratification (weights) into the problem. The sampling strategy assigns weights to the sampling scheme in order to up weight or down weight cases; we call this approach “sampling stratification”.

Pollfish survey platform provides a service for sample stratification by age at a higher cost; the service allows stratification for a prefixed age ranking given by the platform as shown in the first row of Table 2. The sampling proportions in the second row of the table were set on the basis of the population proportions provided by INE official statistics. This sampling stratification strategy is also applied to the designs of Andalusia and Valencia region surveys; the sampling proportions are shown in Table 2.

5 Results

The proposed methodology has been applied to datasets obtained from indirect surveys conducted in the communities of Madrid, Andalusia, and Valencia during the electoral campaigns of 2023 Spanish

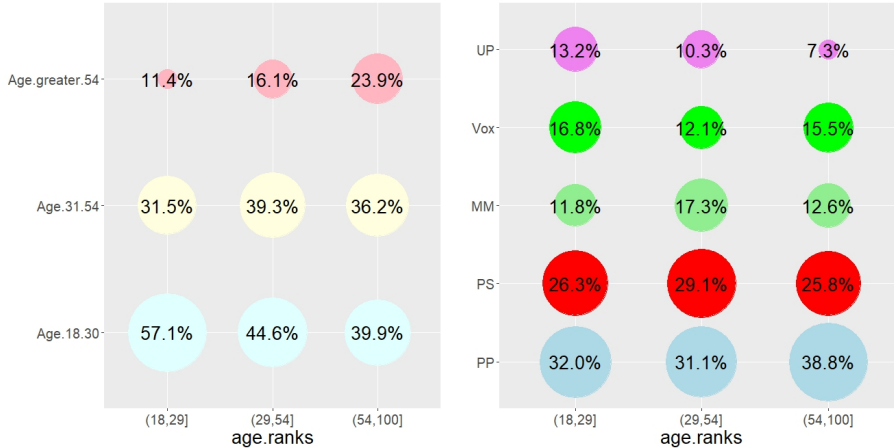


Figure 1: Age declaration from the network of respondents against the age of the respondent (left). Voting declaration from the network of respondents against the age of respondent (right) (Numbers in percentages). The parties’ acronyms are UP-Unidas Podemos, Vox, MM-Más Madrid, PS-Partido Socialista de Madrid, and PP-Partido Popular.

Madrid	Age	[18, 24]	[25, 34]	[35, 44]	[45, 54]	≥ 55
	Sampling prop (%)	8.78	12.32	15.66	20.23	43.01
Andalusia	Age	[18, 24]	[25, 34]	[35, 44]	[45, 54]	≥ 55
	Sampling prop (%)	9.32	12.98	16.43	19.64	41.64
Valencia	Age	[18, 24]	[25, 34]	[35, 44]	[45, 54]	≥ 55
	Sampling prop (%)	8.79	11.65	15.33	19.96	44.26

Table 2: Weights by age for the stratified sampling scheme by region

general elections. Indirect surveys were collected using a stratified sampling strategy, with stratification proportions based on age, as described in Table 2. Furthermore, the results of the various NSUM estimators have been extrapolated to the representative seats of each political party within the Spanish parliament for each region (Madrid, Andalusia, and Valencia). The allocation of representative seats for the political groups was determined using the D’Hondt method, which is explained in the Supplementary Material B. Then, we compare the resulting seat estimations with those published by a famous survey company in Spain and the official estimations launched by the public reference center for sociological studies in Spain. The results are discussed separately for each region.

5.1 Results for the General Elections (Madrid)

An online survey was conducted using the Pollfish platform to collect data for the Spanish general elections held on July 23, 2023. The survey was implemented in the Community of Madrid from July 13 to 21, concluding two days before election day. In particular, our study gathered information on voting intentions and control questions related to the closest contacts of 200 surveyed individuals. The English version of the questionnaire is available in the Supplementary Material, Section A.1.

In the initial endeavor, and for illustrative purposes, we calculated voting intention rates by applying the different NSUM estimators detailed in Table 1 to the raw data. Table SM1 in the Supplementary Material shows the estimated proportions for each NSUM estimator by political party. This table also displays percentages for the conservative (PP+Vox) and progressive (PSOE+Sumar) ideological blocs. This scenario serves as an example that justifies the need to carry out data processing steps. The table includes the percentages of blank votes alongside the official results in Madrid, suggesting that the blank vote is considerably overrepresented. More precisely, the official outcome for the blank vote is 2.9%, while the estimated rates obtained by the Naïve, MoS, MLE, and PIMLE methods are 10.4%, 14.8%, 21.8%, and 14.9%, respectively. Overestimation can be attributed to respondents who misinterpret what a Blank vote means. To improve our estimates, only the filter that removes responses declaring many blank voters above a specific threshold is implemented. The preprocessing stage yielded 151 valid responses, covering 4375 individuals, with an average of approximately 29.0 indirect votes per respondent.

Method	Political parties				Ideological blocs		
	Vox	PP	PSOE	Sumar	PP+Vox	PSOE+Sumar	Blank
Naïve	16.1	35.1	28.1	16.2	51.2	44.3	4.6
MoS	14.9	37.8	27.5	16.1	52.7	43.6	3.7
MLE	15.4	36.3	29.1	16.4	51.7	45.5	2.8
PIMLE	14.6	39.2	27.6	14.7	53.9	43.4	3.8
Real outcome	13.9	40.2	27.7	15.3	54.1	43.0	2.9

Table 3: Spanish general elections, July 2023. Indirect survey collected in the Community of Madrid. Estimation of Percentage of votes by political group and blank vote after filtering out samples that contained a significant number of blank votes. Estimated percentage for the ideological blocs (conservative: PP+Vox and progressive: PSOE+Sumar).

Method	Voting percentage				Seats (D’Hondt method)			
	Vox	PP	PSOE	Sumar	Vox	PP	PSOE	Sumar
GAD3 (July 23)	NA	NA	NA	NA	5	17	10	5
CIS (June 23)	NA	NA	NA	NA	4-5	14-15	10-11	7-8
Naïve (July 23)	16.1	35.1	28.1	16.2	6	14	11	6
MoS (July 23)	14.9	37.8	27.5	16.1	6	15	11	5
MLE (July 23)	15.4	36.3	29.1	16.4	6	14	11	6
PIMLE (July 23)	14.6	39.2	27.6	14.7	5	16	11	5
Real (CERA excluded)	13.9	40.2	27.7	15.3	5	15	11	6
Real (CERA included)	13.9	40.2	27.6	15.3	5	16	10	6

Table 4: Spanish general elections, July 2023. Indirect survey collected in the Community of Madrid. Estimated percentages of votes by political group using the NSUM approaches after filtering out samples that contained a significant number of blank votes. Extrapolation of proportion estimates to representative seats using the D’Hondt method and comparison with other studies. Official results obtained from the Spanish general elections in Madrid, including (CERA included) and excluding (CERA excluded) the votes of Spanish nationals who reside in foreign countries.

Table 3 shows the estimated vote proportions for each political party, derived from applying NSUM methodologies following the aforementioned blank vote filtering process. Additionally, the table presents the estimated voting shares for the conservative (PP+Vox) and progressive (PSOE+Sumar) blocs. Table 3 also displays the official results of the Spanish general elections for the Community of Madrid. Note that the estimated percentages are closer to the real outcomes than [the estimations](#) derived from raw survey data. For example, the blank voting proportion is 2.9%, and the percentages obtained by Naïve, MoS, MLE, and PIMLE estimators are 4.6%, 3.7%, 2.8%, and 3.8%, respectively. Furthermore, we can observe that PIMLE provides the closest estimates to the actual outcome. [Notice that political preferences cannot be considered a “hidden population” in the strict sense. In contrast, these groups can exhibit structured patterns by homophily, geography, demographics, or ideology. To assess the effects of homophilious mixing on voting intention share estimation, the Supplementary Material in Section C presents a sensitivity analysis of the NSUM estimators across different levels of homophily using synthetic graphs.](#)

Finally, Table 4 shows the extrapolation of the NSUM results using the D’Hondt method [BG13] to estimate the number of Madrid representative seats by political group in the Spanish national parliament. For the sake of completeness and comparison, the table also provides the results published by a famous company of surveys in Spain (GAD3, <https://www.gad3.com>) and the official forecast of the national public reference center for sociological studies in Spain (CIS, *Centro de Investigaciones Sociológicas*, <https://www.cis.es/>). The last two rows of Table 4 display the official results excluding (CERA excluded) and including (CERA included) the votes of citizens, registered in the Spanish electoral system, who reside in foreign countries (Censo Electoral de Residentes Ausentes, CERA). We can observe that the PIMLE estimator gives the most accurate seat estimates when compared to other polling [results](#), such as GAD3 and CIS, as well as to other NSUM approaches. For instance, if we consider the PIMLE estimator and the true political party representation outcomes, the conservative bloc obtains 21 seats (PIMLE: PP + Vox = 16 + 5, True: PP + Vox = 16 + 5), and the progressive bloc obtains 16 seats (PIMLE: PSOE + Sumar = 11 + 5, True: PSOE + Sumar = 10 + 6).

Table SM4 in the Supplementary Material presents the vote percentages and the 95% confidence intervals derived from NSUM estimators for the Community of Madrid. The confidence intervals were calculated using an approximate variance method [KMB⁺98a], accounting for the stratified nature of our survey design. Specifically, we first computed the variance for each age group separately. Then, the overall population variance was estimated as a weighted sum of the variances, with the stratification rates in Table 2 serving as the weights. This table also includes the official results. This table indicates that most official results fall within the confidence intervals. The official results only fall outside the confidence intervals for the Naïve approach in the PP and blank options. Furthermore, Table SM5 displays the vote percentages and the 95% confidence intervals obtained from 10,000 respondent-level stratified bootstrap replicates. In each replicate, respondents are resampled within strata, and the full estimation pipeline is rerun, including the preprocessing and vote-share estimation. The resulting vote shares from the bootstrap analysis were then mapped to seats using the D’Hondt rule. Figure SM3 shows the bootstrap distributions of seat allocations by party and NSUM estimator. Concretely, each panel depicts the empirical distribution of the number of seats obtained by a given party under a particular NSUM approach (rows: Naïve, MoS, MLE, and PIMLE; columns: PP, PSOE, Vox, and Sumar). Vertical dashed lines indicate the official outcome.

5.2 Results for the General Elections (Andalusia)

This study conducted an online survey in Andalusia using the Pollfish platform to gather information on voting intention for the Spanish general elections. Specifically, the data collection was performed from July 13 to 17, 2023. In this case, the preprocessing stage produced 137 valid responses, encompassing 2892 individuals, with an average of 21.11 indirect votes per survey participant. To evaluate the impact of the preprocessing steps on the vote share and seat allocation estimates, we present results across a grid of configurations, varying three dimensions: (i) the trimming level applied to the voting declaration network size (80%, 85%, 90%, and 95%), (ii) the threshold used in the median absolute deviation (MAD) rule (3, 4, and 5), and (iii) the inclusion or exclusion of the rule that removes respondents for whom more than 95% of reported alters support the same party. The results are presented in Tables SM10 and SM11 in the Supplementary Material for four estimation methods (Naïve, MoS, MLE, and PIMLE) and are reported in terms of both vote shares and implied seat allocations. Table 5 displays the percentage voting estimations obtained by the NSUM estimators for each political party after applying the data processing criteria described in Section 4.2. We can observe that the Naïve method outperforms the others in estimating the conservative political options. In contrast, the PIMLE approach seems better for estimating the progressive political options and the PP while underestimating Vox results.

The extrapolation of the estimated vote proportions to the national parliament seats from Andalusia using the D’Hondt method is given in Table 6. The official results, both with and without the CERA vote, are provided for completeness; although, in this case, the CERA vote didn’t change the final representation. The NSUM-based estimates of parliamentary seats are compared with the results published by GAD3 and CIS during the campaign. Overall, we can observe that NSUM outcomes are competitive with GAD3 and CIS estimations, with the MLE method given the most accurate results.

Tables SM6 and SM7 in the Supplementary Material present the percentage voting estimates along with 95% confidence intervals using the approximate variance and bootstrap methods, respectively. Moreover, Figure SM4 displays the bootstrap distributions of seat allocations by party and NSUM technique. In this case, the official results of the political parties and ideological blocs fall inside the confidence intervals. However, the NSUM operators significantly overrepresent the blank voting option with confidence intervals larger than official outcomes.

5.3 Results for the General Elections (Valencia)

For our third use case, we deployed an online survey in the Valencian Community using the Pollfish platform to gather information on voting intention for the general elections of July 23, 2023. This survey was deployed from July 13 to 21, 2023; it was launched for individuals who live in this Community as a sampling framework. Pollfish platform provided 134 valid responses from the data preprocessing stage with 1872 potential individuals and 13.97 votes per respondent. In this case of the Valencia, the results are shown in the Tables 7 and 8. The NSUM methods generally understate the progressive option with respect to the conservative one (see Table 7). The method providing the closest results to the real outcome is the PIMLE, followed by the MoS. Regarding the extrapolation of the estimates to determine the representative seats using the D’Hondt method, we can see in Table 8 that CERA inclusion or

Method	Political parties				Ideological blocs		
	Vox	PP	PSOE	Sumar	PP+Vox	PSOE+Sumar	Blank
Naïve	15.2	35.1	28.3	12.6	50.3	40.8	8.9
MoS	13.1	37.1	27.6	13.4	50.2	41.00	8.8
MLE	13.9	35.6	27.6	14.7	49.5	42.3	8.2
PIMLE	12.7	36.6	30.6	12.3	49.3	42.9	7.8
Real outcome	15.3	36.4	33.5	12.0	51.7	45.5	2.8

Table 5: Spanish general elections, July 2023. Indirect survey collected in Andalusia. Estimation of Percentage of votes by political group and blank vote after filtering out responses by using all data cleaning criteria. Estimated percentage for the ideological blocs (conservative: PP+Vox and progressive: PSOE+Sumar).

Method	Voting percentage				Seats (D'Hondt method)			
	Vox	PP	PSOE	Sumar	Vox	PP	PSOE	Sumar
GAD3 (July 23)	NA	NA	NA	NA	6	29-31	21-23	3
CIS (June 23)	NA	NA	NA	NA	5-7	27-28	18-20	8
Naïve (July 23)	15.2	35.1	28.5	12.3	10	24	19	8
MoS (July 23)	13.1	37.1	27.6	13.4	9	25	18	9
MLE (July 23)	13.9	35.6	27.6	14.7	10	24	18	9
PIMLE (July 23)	12.7	36.6	30.6	12.3	8	25	20	8
Real (CERA excluded)	15.3	36.4	33.5	12.0	9	25	21	6
Real (CERA included)	15.3	36.4	33.5	12.0	9	25	21	6

Table 6: Spanish general elections, July 2023. Indirect survey collected in Andalusia. Estimated percentages of votes by political group using the NSUM approaches after filtering out responses using all data cleaning criteria. Extrapolation of proportion estimates to representative seats using the D'Hondt method and comparison with other studies. Official results obtained from the Spanish general elections in Andalusia, including (CERA included) and excluding (CERA excluded) the votes of Spanish nationals who reside in foreign countries.

Method	Political parties				Ideological blocs		
	Vox	PP	PSOE	Sumar	PP+Vox	PSOE+Sumar	Blank
Naïve	16.5	31.8	28.1	14.3	48.3	42.4	9.4
MoS	16.6	32.2	28.3	14.3	48.8	42.6	8.6
MLE	15.6	32.4	27.7	14.1	48.0	41.8	10.3
PIMLE	16.5	32.3	27.7	15.0	48.8	42.7	8.6
Real outcome	15.7	34.9	32.1	15.2	50.6	47.3	2.2

Table 7: Spanish general elections, July 2023. Indirect survey collected in the Valencian Community. Estimation of Percentage of votes by political group and blank vote after filtering out respondents at the top 20% network size who also declare a network voting intention for a specific party above 95%. Estimated percentage for the ideological blocs (conservative: PP+Vox and progressive: PSOE+Sumar).

Method	Voting percentage				Seats (D’Hondt method)			
	Vox	PP	PSOE	Sumar	Vox	PP	PSOE	Sumar
GAD3 (July 23)	NA	NA	NA	NA	5	14	11	3
CIS (June 23)	NA	NA	NA	NA	4	11	11	7
Naïve (July 23)	16.5	31.8	28.1	14.3	6	12	10	5
MoS (July 23)	16.0	32.1	28.8	15.0	6	12	10	5
MLE (July 23)	15.6	32.4	27.7	14.1	6	12	10	5
PIMLE (July 23)	16.5	32.3	27.7	15.0	6	12	10	5
Real (CERA excluded)	15.7	34.9	32.1	15.2	5	13	11	4
Real (CERA included)	15.7	34.9	32.1	15.2	5	13	11	4

Table 8: Spanish general elections, July 2023. Indirect survey collected in the Valencian Community. Estimated percentages of votes by political group using the NSUM approaches after filtering out respondents at the top 20% network size who also declare a network voting intention for a specific party above 95%. Extrapolation of proportion estimates to representative seats using the D’Hondt method and comparison with other studies. Official results obtained from the Spanish general elections in the Valencian Community, including (CERA included) and excluding (CERA excluded) the votes of Spanish nationals who reside in foreign countries.

exclusion does not affect the distribution of the seats in Valencia. Additionally, the NSUM methods correctly estimate the seats of the conservative (Vox+PP) and progressive (PSOE+SUMAR) blocs, with the GAD3 and CIS surveys failing in such estimations, the latter one giving a different majority than the one provided by the real outcome.

Finally, in the Supplementary Material, [Tables SM8 and SM9](#) show the estimated voting intention alongside the 95% confidence intervals [with the approximate variance and bootstrap approaches](#). [The bootstrap distributions of seat allocations by party and NSUM technique are depicted in Figure SM5](#). The official results fall within the confidence interval for nearly all methods and political parties, except for the estimated percentages obtained by the Naïve method for PSOE. Additionally, similar to the results in Andalusia, the NSUM approaches overrepresent the blank voting option, with confidence intervals larger than the official results.

6 Discussion on the Use of the Methodology in the Case Study

In this section, we analyze the use of our methodology in the Case Study to summarize the key findings from its application. Figure 2 contains the steps of the methodology followed in the Case Study, along with the specific tasks/decisions performed in each step. These steps are detailed below.

6.1 Survey Design and Data Collection

In this first step, the following tasks were performed. First, the platform for conducting the surveys was chosen. In our case, the Pollfish platform was chosen because it had broad coverage of the regions under study. Furthermore, this platform allows for the collection of demographic data such as [the age of participants](#).

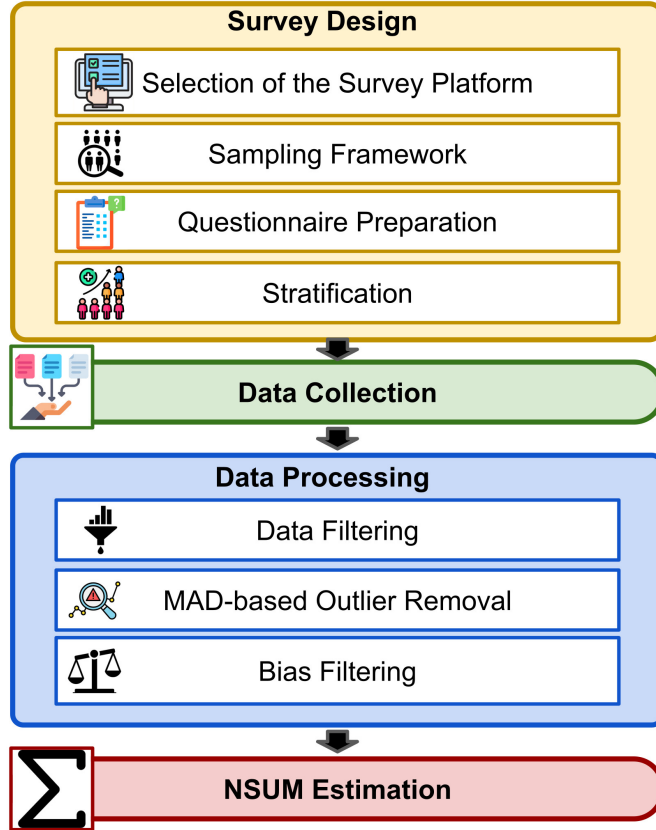


Figure 2: Flowchart of the methodology

For the design of the survey, the following method was used. Each participant was asked: “How many of your contacts will vote for this party for the Spanish general election?” We then added up the responses for each party to obtain the number of contacts. Thus, we defined the network based on the contacts whose voting intentions are known to the participants. This approach minimizes transmission error, since in this case, all contacts have known voting intentions, such that the numerator and denominator refer to the same network. This approach does not compromise the estimates, as long as the hidden subpopulations are evenly distributed in the network [LBN21, DAAR⁺25]. Furthermore, there is another source of transmission error that can affect estimation outcomes, which is that participants misjudge their contacts’ voting intentions, a bias that our framework does not correct for. However, given the overall proximity of our estimates to the actual results, this suggests that the influence of this bias is not significant.

In turn, in the first survey, we observed a barrier effect because younger people were more likely to know people their own age, who in turn were more likely to vote for specific parties. Age stratification is used to mitigate this effect. Although we know that age stratification on its own cannot make contacts composition representative, we use it to reduce the impact of undesirable biases.

Specifically, to obtain a stratified representative sample, age was used in the fourth task because it was provided by the tool and we could easily link it to INE official statistics. If we had another demographic variable with official statistical values, and we could obtain that information from the respondents through the survey platform, such variable could also be used for stratification. Our methodology is not tied to age as a demographic variable for stratification.

Based on everything indicated above, the surveys were collected through the Pollfish platform.

6.2 Data Processing

Three tasks were used for data preparation. The first task was to eliminate 20% of the cases that report the highest number of contacts’ voting intention in their social network, a threshold based on an empirical analysis which uses box plots as mechanisms to detect outliers (Figure 3 shows an example of the box plot for Madrid, with outliers (left) and without outliers (right)). While the box plot method

eliminated 16.6% of the responses in the Madrid survey, 20% in Andalusia and 18.7% in Valencia, a conservative unified criterion was used, and 20% was applied to all. This filter seeks to mitigate the effect of responses with disproportionately large network sizes, which could be due to declaration errors, ambiguous interpretation of the question, or unrepresentative extreme cases that distort the overall analysis. This allows the analysis to focus on the 80% of the responses that, due to their size, are presumably more representative of the phenomenon under study. The declarations’ extremely large voter biases are often outliers that can exert a disproportionate influence on the aggregate results, such as by skewing the mean, reducing representativeness, or introducing “noise” or *atypical* cases that do not reflect the general behavior being studied.

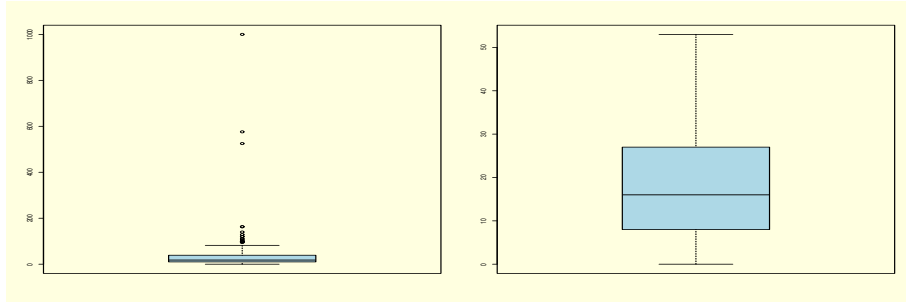


Figure 3: Boxplots of declared network size with (left) and without (right) outliers for Madrid surveys.

The second task was to eliminate responses that reported a network with blank voters above a threshold defined by MAD equal to 5. This method allows for the detection of observations that deviate significantly from the general behavior of the sample. By eliminating these cases, the analysis focuses on networks where there is discernible partisan voting intention, improving the clarity and relevance of the results for the specific objective of studying the distribution of votes between parties. The application of this filter ensures the quality and consistency of the results, minimizing the impact of responses that are potentially unrepresentative, erroneous, or influenced by distorted perceptions of the respondent’s social environment. A high percentage of blank voters within a network can indicate several situations that can distort the analysis of partisan voting intention, such as: (i) Protest or widespread discontent: This could reflect a protest against the current political system, rather than a preference for a particular partisan option, which does not help in the analysis of the distribution of voting intentions among parties and introduces “noise”. (ii) Lack of decision or extreme indecision: The network could be composed of individuals who have not yet decided on their vote or who do not feel represented by any partisan option, which is meaningless in an analysis of voting intentions. (iii) Problems in understanding the question: This could be indicating that participants did not understand the question. (iv) Bias in the composition of the network: It is possible that the network is a subgroup of individuals who tend to vote blank in a greater proportion than the general population, which could bias the results.

In the third task, responses stating that more than 95% of their network would vote for the same party were eliminated, as these types of statements may indicate biased, exaggerated, or unrepresentative responses of the actual social environment. Such responses could distort the analysis, as they do not reflect the typical diversity of opinions on real social networks, thus compromising the validity of the results. This threshold mitigates potential social desirability biases, extreme information bubbles (echo chambers where information within the network may be highly biased toward one political perspective), or errors in self-perception of the political environment (leading to an unrepresentative sample).

6.3 NSUM Estimation

The final stage consists of applying the different NSUM methods, then selecting the one that gave the best results for comparison purposes. Furthermore, in this case, the D’Hondt method was applied to determine the number of representatives per party, as it is the method of electoral representation used by the Spanish electoral system.

7 Conclusions

This paper introduces a comprehensive methodology for estimating the voting outcomes of electoral processes using aggregated relational data (ARD) from indirect surveys. Our proposal encompasses

a unified approach that extends from the design of the indirect survey to the computation of vote estimations by utilizing several NSUM-based procedures. In essence, the proposed methodology contains three main stages: the first one focuses on designing the indirect survey to collect information on voting intention. Notice that this first step considers demographic information to enhance data reliability. Secondly, an ARD data processing step is addressed, with different filtering criteria being proposed and implemented for data cleaning. Finally, our approach estimates voting intention percentages by means of NSUM procedures. The proposed methodology was assessed to estimate the proportions of voting intentions and parliamentary seats in three Spanish regions—namely, the Community of Madrid, Andalusia, and the Valencian Community—for the general elections held on July 23, 2023. The results obtained are promising because they improved on the results reported by various direct surveys in all cases studied. Moreover, our proposal of using indirect surveys and NSUM to estimate the outcomes of elections has the advantage of reducing the costs of standard direct surveys, so it is expected to draw the attention of private and public poll organizations, providing an alternative to their survey plannings.

The use cases developed for the general 2023 elections in Madrid, Andalusia, and Valencia demonstrate the potential of applying the proposed [NSUM-based](#) approach. In these cases, the survey was designed in accordance with a sample stratification scheme by age, with strata proportional to the ages of the voting adult population. When the proposed NSUM-based procedures are applied to estimate party voting proportions, their outputs provided promising results compared with those provided by public and private polling institutions, based on direct reporting surveys. In some instances, the proposed methodology highly surpasses traditional direct reporting estimations. Finally, it is worth noting about sample sizes for all the Spanish territory of GAD3 ($n = 3502$) and CIS ($n = 29201$) surveys. Thus, for example, in the case of GAD3, the number of samples from the smallest region considered (Community of Valencia, with a voting population of slightly more than 3.6 million, out of a Spanish voting population of 35 million) can be estimated to be at least $3502 \times 3.6/35 = 360$, almost twice our sample size. In the case of CIS, these sample sizes are even larger.

One thing learned from this study is that sampling stratification may have an impact on NSUM resulting outcomes. To illustrate this, we conducted an initial analysis that did not consider stratification when collecting samples and then used weighted estimation (post-stratification) as an alternative method. This initial methodology was applied to the autonomic elections that took place in Madrid on May 28th, 2023 (regional elections). The application of each step of the post-stratification methodology is described in the Supplementary Material A.3. The resulting estimations are worse than those ones obtained by applying the proposed sampling stratification strategy. Hence, we advocate the use of sampling stratification from the very beginning as it provides more accurate results. Summing up, the combination of sampling stratification and NSUM-based approaches provides better results than those obtained by GAD3 and CIS survey institutions from direct reporting and the ones obtained with the NSUM without sampling stratification. The previous findings open the door to future research, which would address a deep analysis of different sampling strategies to choose the strategy that guarantees a representative sample for any electoral process (see [Section 3.1](#), where we make a first comment on possible strategies). In particular, future work will evaluate alternative stratification sampling strategies using other demographic variables, which would be possible provided that these variables are available both in the survey platform and in the official statistics. [Additionally, NSUM estimators are affected by the homophily among supporters of different political parties. Another direction for future work is the estimation of homophily using ARD and the development of correction of the NSUM estimators.](#)

Although indirect surveys have advantages in terms of implementation costs and the [protection](#) of respondents and their contacts, our proposal has some limitations. The first drawback concerns the use of technological platforms to design and collect indirect surveys, which usually under-represent populations unfamiliar with digital environments or over-represent people accustomed to digital survey tools. More research is needed on sampling design to address this technological bias. A second limitation is that, although our methodology has been tested in different regions of Spain, such as Madrid, Valencia and Andalusia, and the results are encouraging, we believe more field results are necessary in other countries and electoral contexts (such as the European elections) to evaluate the validity and reproducibility of the proposal. Hence, future work should include applying the methodology to other electoral processes, exploring in detail the role played by different sampling strategies, survey design and weighting estimation mechanisms in the implementation of the NSUM and the resulting outcomes when deployed within different electoral systems. The third issue for future research would involve the study of alternative data processing filters that could be incorporated into the methodology to improve the accuracy of the estimations. Another limitation is the assumption we make when we consider that respondents know the voting intentions of their circle of acquaintances and that they do not incur any political bias due

to their voting intentions or due to uncertain knowledge about the voting intentions of their contacts (transmission error). This may be one of the causes of the overestimation of blank votes. Future work should analyze these potential biases in depth.

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Ethical Approval

Participants were recruited through Pollfish and provided informed consent prior to participation. The consent explained the purpose of the study and the voluntary nature of participation. To minimize potential risks for respondents, we have designed the survey such that personal information is not required, asking questions solely about the contacts of the participants. It is worth noting that this information cannot identify users. However, captured data is processed by Pollfish as personal data according to the EU regulation for the protection of personal data (GPDR), as can be seen on its webpage, and according to the laws applicable to the users of this platform. Pollfish also claims to comply with applicable data protection law when receiving personal data from participants, i.e., respondents have been informed about Pollfish privacy terms, and the compliance with the requirements of applicable law since they signed up for the Pollfish platform and agreed to the terms of use (see <https://www.pollfish.com/terms/respondent>). The participants have also been informed about the data, how they use it, where it goes, and how long it is kept. The raw data available in Pollfish and provided to the researchers includes, in addition to the responses to the survey questions, a number of other attributes of each participant. We did not use the raw data, but instead we extracted the following attributes provided by Pollfish: gender, year of birth, education (5 options), income (8 intervals), and employment status (10 options). The raw data were immediately deleted after this process. Data were stored on institutional secure storage, accessible only to the research team. Ethics approval was obtained from the Institutional Review Board (IRB) of the IMDEA Networks Institute, which approved this research in May and July 2023 for the regional and general elections, respectively.

Availability of Data and Materials

Data presented in this paper in aggregated form, as well as the codes used to process the data, can be provided by the authors upon a reasonable request.

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A Supplementary Material

A.1 Questionnaire for the General Election in Madrid, Andalusia and Valencia

- **Q1. Scientific Study Information**

We are a team of computer and statistics researchers. We collect data to estimate the voting intention in different Spain regions, with the goal of improving statistical estimation techniques. The following survey contains questions regarding the voting intention of the people that you know for the Spanish general election on July 23, 2023. These contacts can include family, friends, and colleagues whose voting intention you know.

- Responses are anonymous. The researchers cannot identify the participants.
- Data cannot be modified or deleted once the survey is completed.
- The data collected may be made public in aggregated form under the *Open Data Commons Attribution License*.
- The results of this study will not be published before the end of the Spanish general elections on July 23, 2023.

You can participate in the survey even if you have answered it before.

Are you willing to take part in the following survey?

- Yes, I'm above the age of 18, and I'm giving you my consent.
- No, I do not wish to take part.

Consider only your contacts of voting age in Spain's general election on 23 July 2023.

- **Q2.** How many of your contacts will vote for PP (Partido Popular) for the Spanish general election?
- Q3.** How many of your contacts will vote for PSOE (Partido Socialista Obrero Español) for the Spanish general election?
- Q4.** How many of your contacts will vote for Vox for the Spanish general election?
- Q5.** How many of your contacts will vote for Sumar for the Spanish general election?
- Q6.** How many of your contacts will vote for other parties or blank for the Spanish general election?
- Q7.** How many of your contacts will not vote or you do not know their vote for the Spanish general election?
- Q8.** How many of your contacts considered above are between 18 and 34 years old?
- Q9.** How many of your contacts considered above are between 35 and 54 years old?
- Q10.** How many of your contacts considered above are over 54 years old?
- Q11.** How many of your contacts considered above are medical doctors?
- Q12.** How many of your contacts considered above are autonomous workers?
- Q13.** How many of your contacts considered above are unemployed?

A.2 Questionnaire for the Regional Election in Madrid

- **Q1. Scientific Study Information**

We are a team of computer and statistics researchers. We collect data to estimate the **voting intention** in different Spain regions, with the goal of improving statistical estimation techniques. The following survey contains questions regarding the voting intention of the people that you know. These contacts can include **family, friends, and colleagues** whose voting intention you know.

- Responses are anonymous. The researchers cannot identify the participants.
- Data cannot be modified nor deleted once the survey is completed.
- The data collected may be made public in aggregated form under the *Open Data Commons Attribution License*.

Method	Political parties				Ideological blocs		
	Vox	PP	PSOE	Sumar	PP+Vox	PSOE+Sumar	Blank
Naïve	15.4	32.1	27.0	15.2	47.5	42.2	10.4
MoS	14.0	31.2	25.5	14.5	45.2	40.0	14.8
MLE	14.4	31.7	20.6	11.5	46.1	32.1	21.8
PIMLE	13.6	32.8	25.0	13.7	46.4	38.7	14.9
Real outcome	13.9	40.2	27.7	15.3	54.1	43.0	2.9

Table SM1: Spanish general elections, July 2023. Indirect survey collected in the Community of Madrid. Estimation of Percentage of votes by political group and blank vote from the raw dataset. Estimated percentage for the ideological blocs (left-wing: PSOE+Sumar and right-wing: PP+Vox).

- The results of this study will not be published before the end of the regional elections in Spain on May 28, 2023.

Are you willing to take part in the following survey?

Yes, I'm above the age of 18, and I'm giving you my consent.

No, I do not wish to take part

Note: 50% of the participants will read Text A and the other 50% will read Text B

Text A

Consider only your contacts who are of voting age in the Community of Madrid.

Text B

Consider only your 20 closest contacts who are of voting age in the Community of Madrid.

- **Q2.** How many of your contacts **have not decided their vote** for the election to the Assembly of the Community of Madrid?
- **Q3.** How many of your contacts will vote for **PP (Partido Popular)** for the election to the Assembly of the Community of Madrid?
- **Q4.** How many of your contacts will vote for **Más Madrid** for the election to the Assembly of the Community of Madrid?
- **Q5.** How many of your contacts will vote for **PSOE-M (Partido Socialista Obrero Español de la Comunidad de Madrid)** for the election to the Assembly of the Community of Madrid?
- **Q6.** How many of your contacts will vote for **Vox** for the election to the Assembly of the Community of Madrid?
- **Q7.** How many of your contacts will vote for **Unidas Podemos** for the election to the Assembly of the Community of Madrid?
- **Q8.** How many of your contacts **will not vote** for the election to the Assembly of the Community of Madrid?
- **Q9.** How many of your contacts **will vote blank** for the election to the Assembly of the Community of Madrid?
- **Q10.** How many of your contacts considered above are **between 18 and 30 years old**?
- **Q11.** How many of your contacts considered above are **between 31 and 54 years old**?
- **Q12.** How many of your contacts considered above are **over 54 years old**?
- **Q13.** How many of your contacts considered above are **medical doctors**?
- **Q14.** How many of your contacts considered above are **autonomous workers**?
- **Q15.** How many of your contacts considered above are **unemployed**?

Method	Political parties					Ideological blocs		
	Vox	PP	PSM	MM	UP	PP+Vox	PSM+MM+UP	BLANK
Naïve	16.3	33.3	21.3	11.8	9.7	49.6	42.8	7.6
MoS	16.3	36.5	19.8	10.6	10.0	52.8	40.4	6.8
MLE	12.4	29.8	17.7	19.1	5.6	42.2	42.4	15.4
PIMLE	15.8	35.6	19.8	11.1	10.6	51.4	41.5	7.1
Real outcome	7.3	47.3	18.2	18.4	4.7	54.6	41.3	4.1

Table SM2: Regional elections: Indirect survey collected in the Community of Madrid. Estimation of Percentage of votes by political group and blank vote from the raw dataset. Estimated percentage for the ideological blocs (left-wing: PSM+MM+UP and right-wing: PP+Vox).

A.3 Results of the Regional Election in Madrid

A.3.1 Survey design and data collection

Below, we describe how the information has been obtained and how the survey has been designed for this electoral process.

- Three rounds of surveys were conducted on the third, second, and last week before the voting day for the regional elections.
- Initially, two situations are considered when asking the respondents: the first one is known as the fix-reach scenario as the surveyed individual is asked to think about a given number of known contacts. In the second situation, we do not impose the limitation of the number of contacts of surveyed individuals so it is left open for him/her to specify the size of his/her network (without reach scenario). Particularly, we consider a fixed-reach scenario with the 20 closest contacts and another scenario without reach. The reported data showed many inconsistencies in the fix-reach scenario, so we decided to work only with the without-reach scenario.
- **Sampling framework:** The surveyed people are individuals who live in the Madrid region, are at least 18 years old, and have a potential voting network also in the Madrid region. Moreover, the respondents are asked only about contacts in their social network, belonging to Madrid regional electoral census.
- **Sample size:** $n = 100$ and no stratification.
- **Target unknown populations:** The main political groups presenting candidates for the regional elections: Partido Popular (PP), Partido Socialista de Madrid (PSM), Vox, Más Madrid (MM), and Unidas Podemos (UP); hence, $Q = 6$ as we are considering the blank vote as an alternative option.
- **Auxiliary known populations:** We have made the same considerations as in the national election (see previous section).
- **Questionnaire:** Similar to the [questionnaire of the general election](#) (see Appendix A.2), where [target and auxiliary data sources](#) are collected from respondents by asking “How many people who ... do you know?” questions.

A.3.2 Data processing

Similar to the [general](#) election, we establish the same rules for the assessment and elimination of outliers and “dirty” information. Based on these rules, the same scenarios were considered. For the sake of completeness, the appendix with the Supplementary Material also includes tables (see Table SM2) displaying the estimations that result from the application of NSUM procedures when no filtering is considered. Just to illustrate the number of surveys removed after data cleaning, we can observe that the simultaneous application of the previous rules would lead to 71 surveys for the regional elections.

Due to the existence of *barrier effects*, the networks surrounding the participants are not representative. We must introduce some ad hoc adjustments based on post-stratification (weighted estimation) in order to address the lack of representativeness and limit the impact of the barrier effect. The next

section deals with the application of NSUM within the framework of stratification strategies to solve this problem.

A.3.3 NSUM-based estimation procedures

The biases that may arise from survey data collection, can also be handled by an alternative strategy that relies on the idea of introducing weights to compute the estimations. The proposal is a weighted estimator, which is calculated by weighting NSUM estimations by age. This approach can be understood as a type of post-stratification that up-weights or down-weights the NSUM estimated voting proportions by age. As in our case we have three age ranges, we compute NSUM estimators $\hat{p}_{uj} : j = 1, 2, 3$ by each range age, and weight them as follows:

$$\hat{p}_{uw} = w_1 \cdot \hat{p}_{u^1} + w_2 \cdot \hat{p}_{u^2} + w_3 \cdot \hat{p}_{u^3} \quad (\text{SM5})$$

The weights of the weighted estimator above must be set in advance so that the ages under-represented in the sample can be up-weighted and the over-represented ones can be down-weighted.

Let us denote by $w^p = (w_1^p, w_2^p, w_3^p)$ the vector containing proportions by age in the general population and let us put $w^s = (w_1^s, w_2^s, w_3^s)$ to denote the same vector in the sample. The components of w^p can be obtained from official statistics. Then, we can set the weights in (SM5) as follows:

$$w_j = \frac{w_j^p/w_j^s}{\sum_{j=1}^3 w_j^p/w_j^s} : j = 1, 2, 3. \quad (\text{SM6})$$

The insertion of the weights above into the expression of the weighted estimator in (SM5), together with the application of the NSUM approaches from Table 1 by age range, will lead to different versions of the NSUM weighted estimation: \hat{p}_{uw}^{NAIVE} , \hat{p}_{uw}^{MoS} , \hat{p}_{uw}^{MLE} and \hat{p}_{uw}^{PIMLE} .

A.3.4 Results for the regional elections

This methodology is applied to the data coming from the indirect surveys carried out in the Madrid autonomic (regional) elections of 2023. Specifically, an online survey was conducted using the Pollfish platform to gather data on the regional election in the Community of Madrid that took place on May 28, 2023. The questionnaire was written in Spanish; its English version is provided in the Supplementary Material, Section A.2. The Community of Madrid was chosen as study case because it has a single province electoral district. In the Supplementary Material, Figs SM1 and SM2 display the demographic characteristics of the survey respondents extracted from the raw data and the samples retained after the filtering preprocessing stage described above. Observe that for the age groups of the filtered data, the proportions of respondents are: 18-24: 21.9%, 25-34: 21.9%, 35-44: 17.8%, 45-54: 19.2%, and >54: 19.2.2%. These proportions are different from those provided by INE [official statistics](#). Therefore, we perform the weighting estimation procedure to adjust the target population estimates. The survey data reported that the average indirect votes from respondents' network is 19.9; as the data processing step has led to 71 valid responses, we definitely handle around 1420 records by means of an indirect declaration of voting intention.

The population proportions by age range in the region of Madrid are $w^p = (0.15, 0.42, 0.43)$ (data provided by INE [official statistics](#)); whereas in the sample they are $w^s = (0.23, 0.55, 0.22)$. As we can observe, the older segment is under-represented in the survey which may enhance the barrier effects highlighted in Figure 1. Hence, the weighted estimator proposed in (SM5) —with the weights being adjusted by means of (SM6) to handle both under or over-representation— is employed to get the voting proportion estimations. In this case, the weights are $w = (0.20, 0.23, 0.57)$.

The computation of the proposed weighted estimator with the NSUM estimation procedures summarized in Table 1 gives the voting estimation outcomes of Table SM3. The table displays the estimations obtained for each political group, from the conservative parties, Vox and PP, to the progressive ideological ones PSM, MM, and UP, as well as the BLANK alternative used to capture the blank vote or the voting intention for other minority groups. Finally, the last row reports the true outcome of the elections.

The results show that the proposed approach tends to overestimate Vox and underestimate PP in the conservative bloc; meanwhile, PSM and UP are overestimated and MM is underestimated in the progressive bloc; note that the BLANK alternative is overestimated. These results may be explained by the recall errors effect [LBN21] as people can usually figure out the ideological profile of the contacts in their network rather than the specific political group each contact will vote for. The issue gets clarified

Method	Political parties					Ideological blocs		
	Vox	PP	PSM	MM	UP	PP+Vox	PSM+MM+UP	BLANK
Naïve	14.0	34.2	21.1	12.7	11.0	48.2	44.8	7.0
MoS	15.2	34.6	19.8	12.9	11.0	49.8	43.7	6.5
MLE	12.3	40.2	20.5	11.6	9.4	52.5	41.5	6.0
PIMLE	14.3	35.3	20.4	12.2	10.8	49.6	43.4	7.1
Real outcome	7.3	47.3	18.2	18.4	4.7	54.6	41.3	4.1

Table SM3: Estimation of Percentage of votes by political group for the regional elections.

when the outcomes are organized and gathered as appearing on the right side of the table: PP+Vox (conservative bloc), PSM+MM+UP (progressive bloc with the latter two allied in Sumar for the national elections). Now, the results provided by the weighted NSUM procedure are closer to the true outcome of the regional elections, with the MLE method beating the other ones.

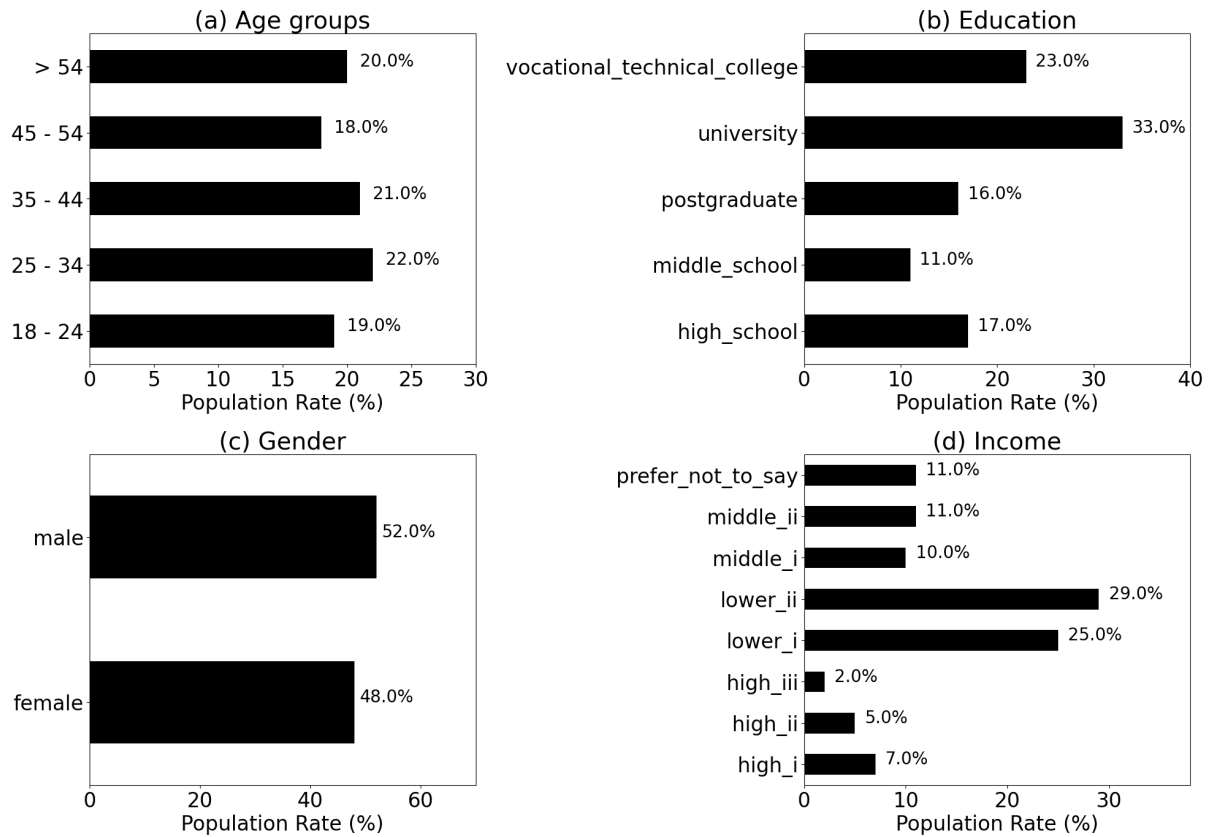


Figure SM1: Madrid regional elections, May 2023. Indirect survey collected in the Community of Madrid. Demographics extracted from raw data. (a) Age groups, (b) Education (c) Gender, and (d) Income.

A.4 Tables with NSUM-based Confidence Intervals

The following tables present the results we have obtained by applying NSUM methods, along with their respective confidence intervals (see Tables SM4, SM6 and SM8).

Method	Political parties			
	Vox	PP	PSOE	Sumar
Naïve	16.06 (12.73-19.38)	35.07 (30.75-39.39)	28.07 (24.45-31.68)	16.23 (12.84-19.63)
MoS	14.90 (12.21-17.59)	37.81 (30.35-45.27)	27.47 (23.94-31.01)	16.08 (11.75-20.42)
MLE	15.42 (9.31-21.54)	36.29 (28.07-44.50)	29.09 (24.54-36.65)	16.37 (10.40-22.35)
PIMLE	14.63 (12.06-17.20)	39.24 (31.19-47.28)	27.65 (24.22-31.07)	14.71 (11.97-17.44)
Real outcome	13.9	40.2	27.7	15.3

Method	Ideological Blocs		
	PP+Vox	PSOE+Sumar	Blank
Naïve	51.13 (46.03-56.22)	44.29 (39.30-49.30)	4.57 (3.19-5.96)
MoS	52.71 (44.80-60.61)	43.56 (36.93-50.19)	3.74 (2.84-4.63)
MLE	51.71 (44.15-59.27)	45.47 (38.52-52.41)	2.82 (0.11-5.54)
PIMLE	53.87 (45.42-62.31)	42.35 (37.59-47.12)	3.78 (2.83-4.73)
Real outcome	54.1	43.0	2.9

Table SM4: Spanish general elections, July 2023. Indirect survey collected in the Community of Madrid. Estimation of Percentage of votes by political group and blank vote and its 95% confidence interval after filtering out samples that contained a significant number of blank votes. Estimated percentage for the ideological blocs (conservative: PP+Vox and progressive: PSOE+Sumar).

Method	Voting Intention				
	Vox	PP	PSOE	Sumar	Blank
Naïve	15.70 (12.70-19.36)	33.75 (29.66-38.55)	27.59 (24.19-31.45)	15.75 (12.70-19.36)	5.53(3.78-11.22)
MoS	14.18 (11.16-18.33)	34.85 (27.61-43.25)	26.78 (21.59-31.79)	15.39 (11.42-20.56)	4.77 (3.14-21.77)
MLE	14.98 (9.37-21.11)	34.76 (23.74-43.19)	26.86 (15.80-34.36)	14.72 (8.52-20.67)	3.44 (2.41-34.95)
PIMLE	14.17 (11.05-17.94)	36.45 (28.76-45.32)	26.73 (21.72-31.83)	14.27 (10.99-18.16)	4.67 (3.07-21.02)
Real outcome	13.9	40.2	27.7	15.3	2.9

Table SM5: Spanish general elections, July 2023. Indirect survey in the Community of Madrid. Bootstrap voting intention shares by political group and blank vote, and its 95% confidence interval.

Method	Political parties			
	Vox	PP	PSOE	Sumar
Naïve	15.25 (12.02-18.48)	35.07 (30.99-39.16)	28.52 (24.70-32.36)	12.29 (9.01-15.58)
MoS	13.09 (10.96-15.22)	37.10 (32.69-41.51)	27.55 (24.39-30.72)	13.44 (10.12-16.76)
MLE	13.91 (8.90-18.93)	35.58 (28.27-42.89)	27.59 (21.45-33.74)	14.69 (9.81-19.58)
PIMLE	12.68 (9.89-15.46)	36.59 (28.38-44.79)	30.63 (20.96-40.31)	12.31 (8.89-15.73)
Real outcome	15.3	36.4	33.5	12.0

Method	Ideological Blocs		
	PP+Vox	PSOE+Sumar	Blank
Naïve	50.32 (45.17-55.47)	40.82 (35.59-46.06)	8.85 (6.81-10.89)
MoS	50.19 (44.99-55.39)	40.99 (35.77-46.21)	8.82 (7.03-10.61)
MLE	49.49 (41.27-57.72)	42.29 (34.87-49.70)	8.22 (4.70-11.74)
PIMLE	49.26 (39.15-59.37)	42.94 (32.48-53.41)	7.80 (5.97-9.63)
Real outcome	51.7	45.5	2.8

Table SM6: Spanish general elections, July 2023. Indirect survey collected in Andalusia. Estimation of Percentage of votes by political group and blank vote and its 95% confidence interval after filtering out samples that contained a significant number of blank votes. Estimated percentage for the ideological blocs (conservative: PP+Vox and progressive: PSOE+Sumar).

Method	Voting Intention				
	Vox	PP	PSOE	Sumar	Blank
Naïve	15.18 (12.04-18.47)	35.19 (31.15-39.42)	28.61 (24.90-32.57)	11.99 (8.83-15.41)	8.87 (6.74-11.34)
MoS	13.26 (10.33-16.38)	36.86 (32.07-41.76)	27.71 (23.89-31.74)	12.67 (8.86-17.01)	9.30 (6.80-12.31)
MLE	13.72 (10.69-17.04)	35.96 (31.18-41.05)	27.90 (23.73-32.36)	13.96 (9.45-18.82)	8.22 (6.18-11.08)
PIMLE	12.66 (10.13-15.61)	36.68 (32.25-40.93)	30.55 (25.40-35.82)	11.98 (7.87-16.83)	7.89 (5.35-10.92)
Real outcome	15.3	36.4	33.5	12.0	2.8

Table SM7: Spanish general elections, July 2023. Indirect survey in Andalusia. Bootstrap voting intention shares by political group and blank vote, and its 95% confidence interval.

Method	Political parties			
	Vox	PP	PSOE	Sumar
Naïve	16.46 (13.12-19.80)	31.77 (27.78-35.76)	28.07 (24.22-31.92)	14.25 (10.88-17.63)
MoS	16.02 (13.23-18.81)	32.07 (28.27-35.88)	28.75 (25.38-32.13)	15.01 (12.20-17.81)
MLE	15.60 (7.75-23.45)	32.37 (21.16-43.58)	27.67 (17.98-37.36)	14.10 (6.59-21.61)
PIMLE	16.46 (10.65-22.27)	33.32 (24.89-39.76)	27.65 (21.65-33.65)	14.95 (9.60-20.30)
Real outcome	15.7	34.9	32.1	15.2

Method	Ideological Blocs		
	PP+Vox	PSOE+Sumar	Blank
Naïve	48.24 (42.91-53.57)	42.32 (37.37-47.27)	9.44 (6.88-12.00)
MoS	48.09 (42.71-53.48)	43.76 (39.08-48.44)	8.15 (6.43-9.87)
MLE	47.97 (35.22-60.72)	41.77 (30.18-53.37)	10.26 (3.49-17.02)
PIMLE	48.78 (37.69-59.88)	42.61 (34.10-51.12)	8.61 (5.00-12.22)
Real outcome	50.6	47.3	2.2

Table SM8: Spanish general elections, July 2023. Indirect survey collected in the Valencian Community. Estimation of Percentage of votes by political group and blank vote and its 95% confidence interval after filtering out samples that contained a significant number of blank votes. Estimated percentage for the ideological blocs (conservative: PP+Vox and progressive: PSOE+Sumar).

Method	Voting Intention				
	Vox	PP	PSOE	Sumar	Blank
Naïve	16.28 (13.07-19.81)	32.09 (28.18-36.16)	28.14 (24.38-32.03)	14.17 (10.98-17.69)	9.21 (6.67-11.55)
MoS	15.80 (12.36-19.50)	32.24 (27.88-36.75)	28.56 (24.73-32.63)	14.83 (11.31-18.77)	8.44 (6.03-10.86)
MLE	15.25 (12.06-18.93)	32.97 (28.28-38.03)	27.77 (23.82-32.11)	13.96 (10.68-17.70)	9.85 (6.44-12.82)
PIMLE	16.28 (12.71-20.03)	32.56 (28.09-37.09)	27.70 (24.04-31.64)	14.88 (11.31-18.83)	8.47 (6.03-10.92)
Real outcome	15.7	34.9	32.1	15.2	2.2

Table SM9: Spanish general elections, July 2023. Indirect survey in the Valencian Community. Bootstrap voting intention shares by political group and blank vote, and its 95% confidence interval.

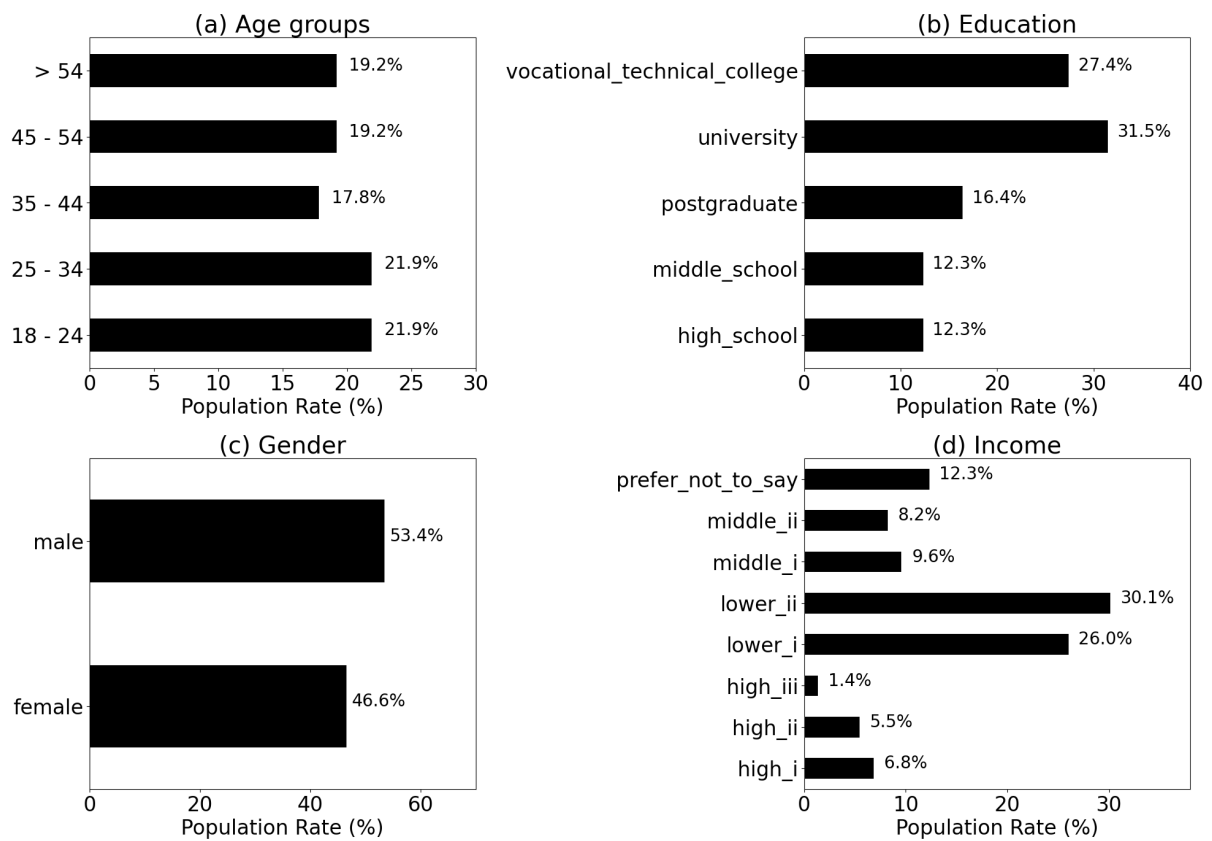


Figure SM2: Madrid regional elections, May 2023. Indirect survey collected in the Community of Madrid. Demographics of the survey respondents after filtering out using the criteria of Section 3.2. (a) Age groups, (b) Education, (c) Gender, and (d) Income.

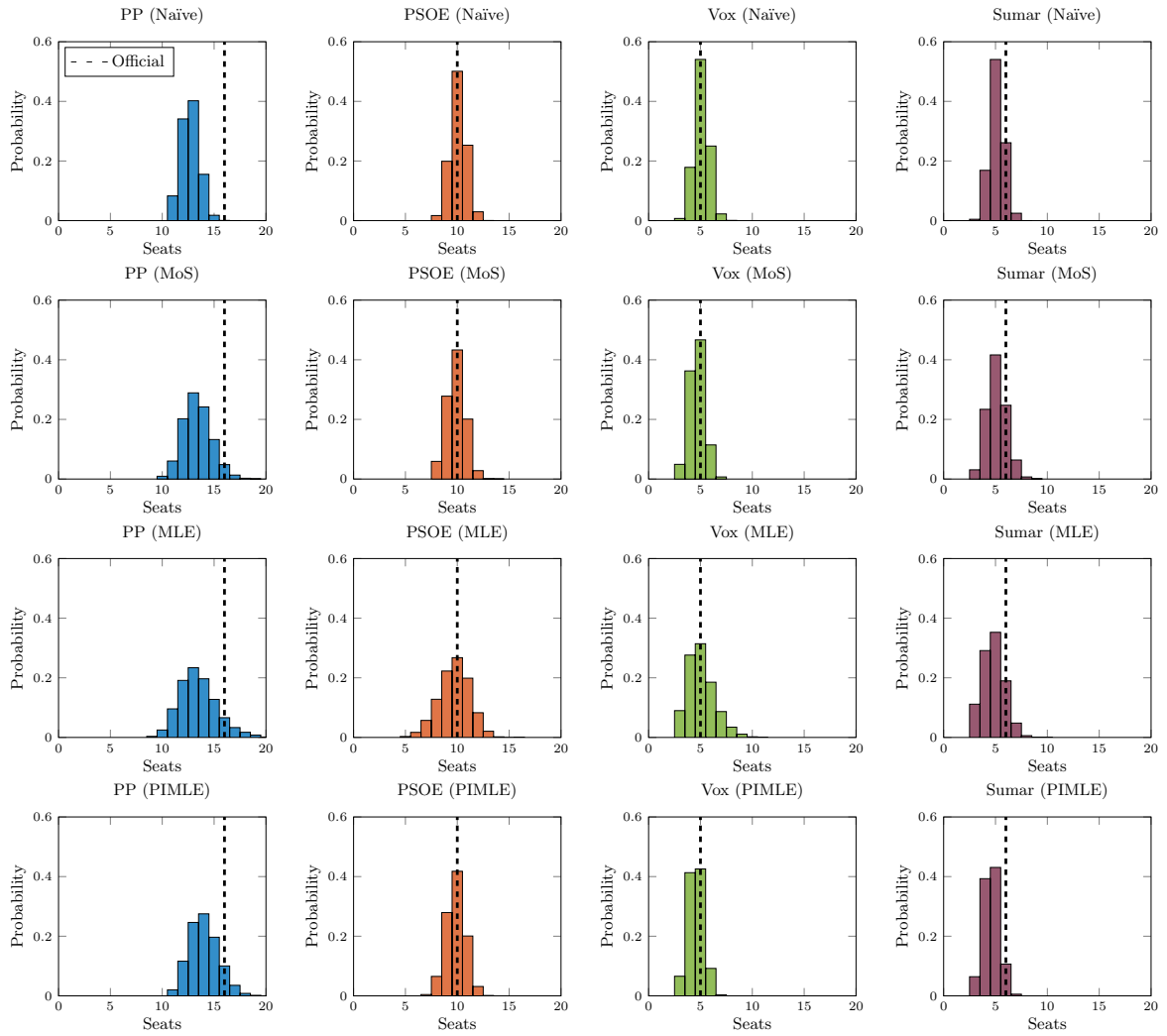


Figure SM3: Bootstrap distributions of seat allocations by party and estimation method. Community of Madrid. Each panel shows the empirical distribution of the number of seats a given party obtains under a specific estimation method.

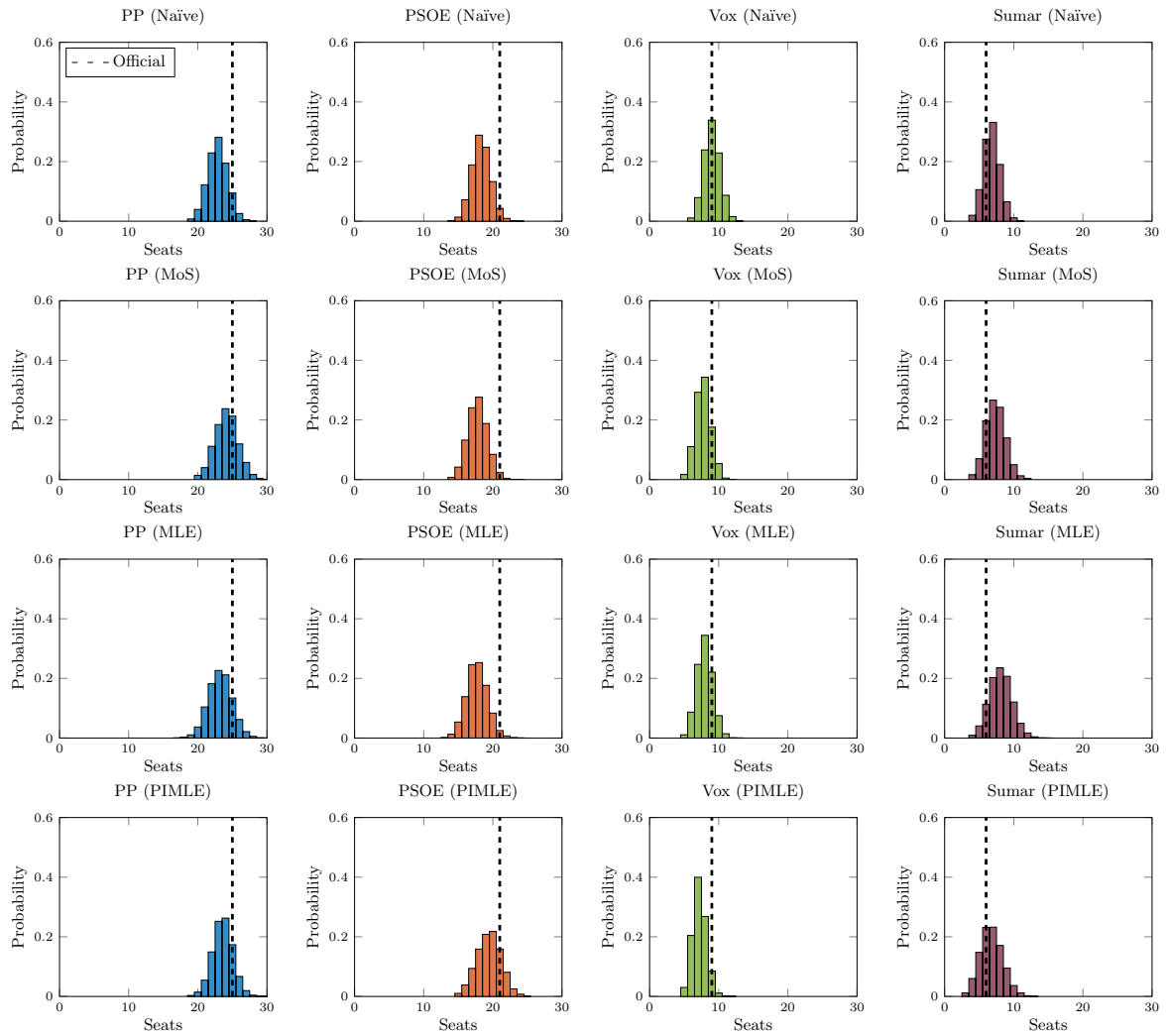


Figure SM4: Bootstrap distributions of seat allocations by party and estimation method. Andalusia. Each panel shows the empirical distribution of the number of seats a given party obtains under a specific estimation method.

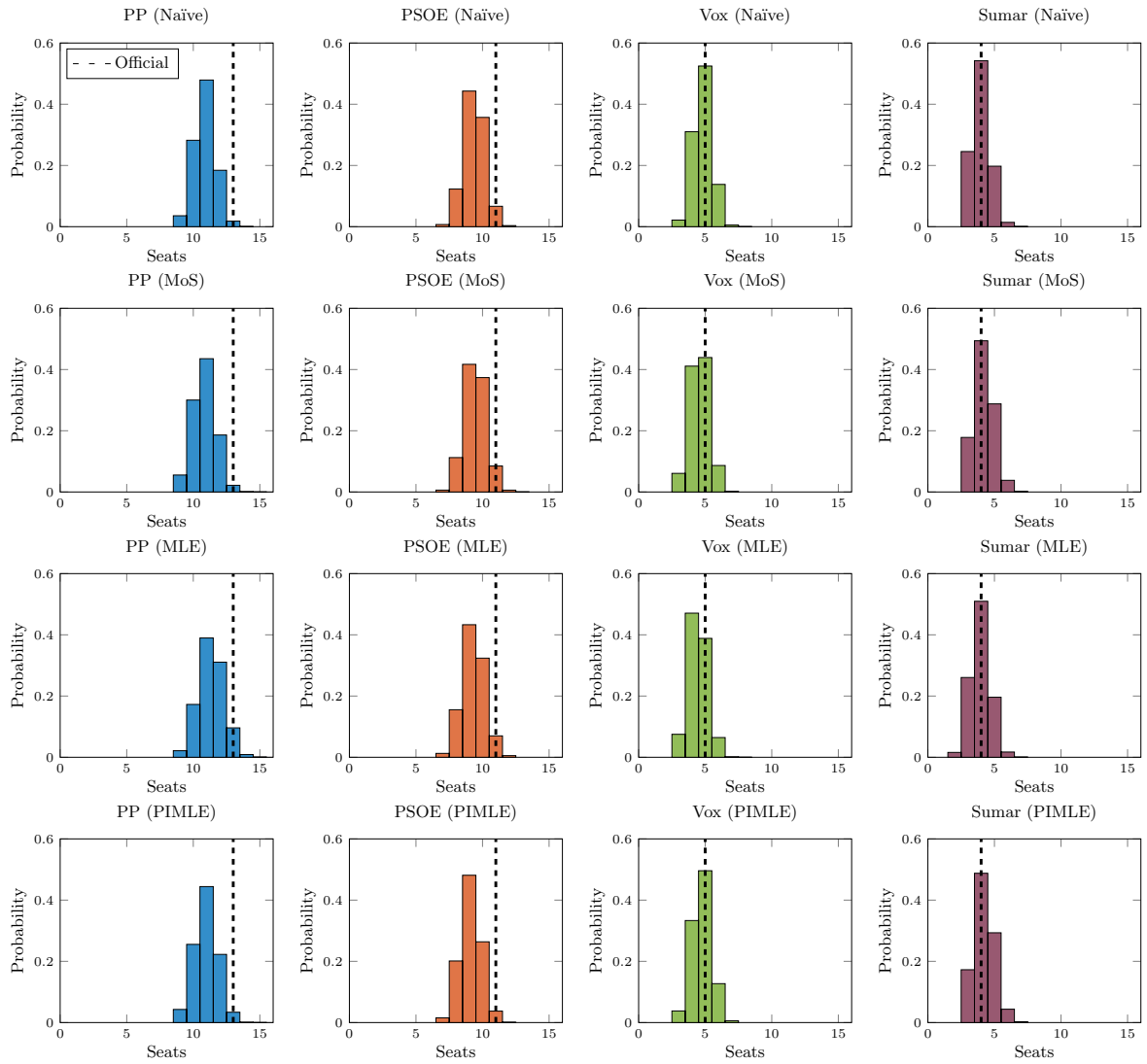


Figure SM5: Bootstrap distributions of seat allocations by party and estimation method. Valencian Community. Each panel shows the empirical distribution of the number of seats a given party obtains under a specific estimation method.

B D'Hondt Method

The D'Hondt method, also known as the process of greatest divisors, allocates seats in parliaments in federative countries and aims for proportional representation among political parties. It is calculated as follows: once all votes have been counted, successive quotients are determined for each party. The party with the highest quotient wins a seat, and its quotient is recalculated. This process continues until the required number of seats is filled. Each quotient is calculated using the formula $quotient = V/(s + 1)$, where V represents the total number of votes received by that party, and s is the number of seats that party has won so far, initially 0 for all parties.

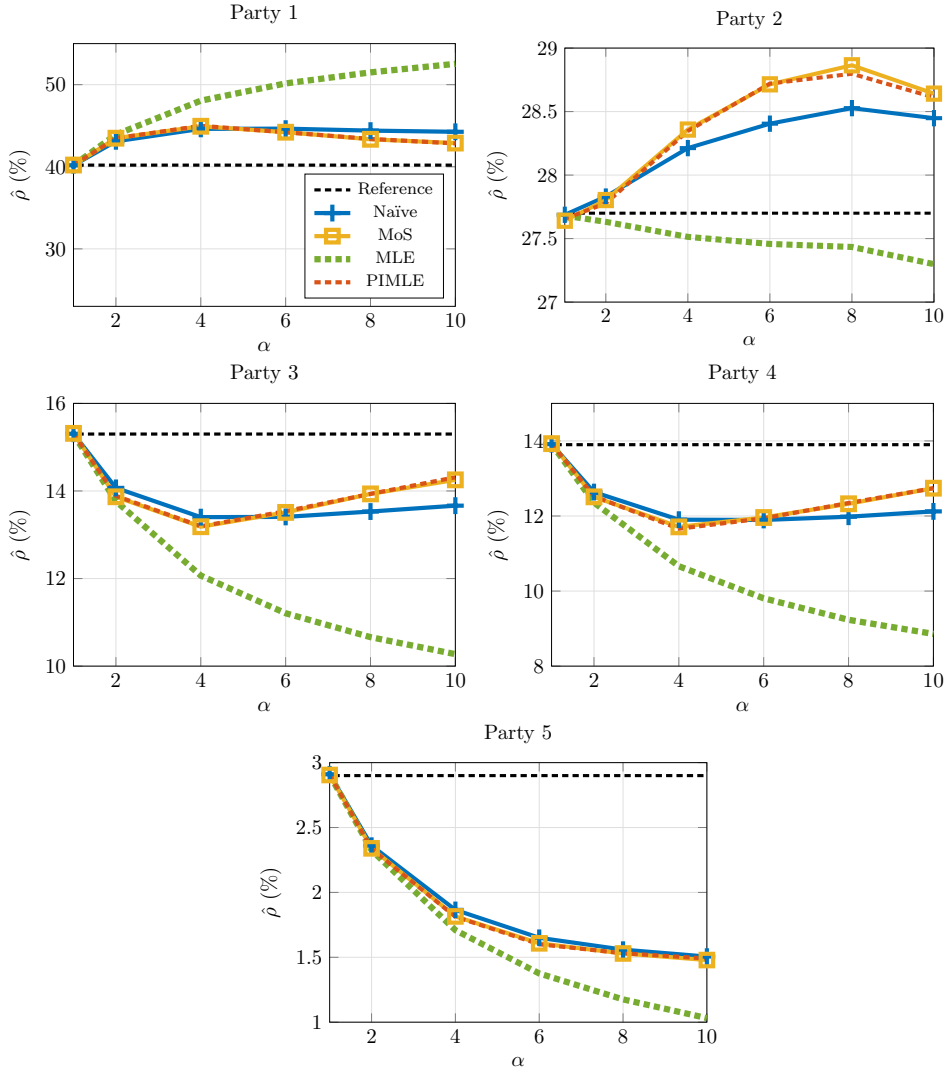


Figure SM6: Estimated prevalence $\hat{\rho}$ in % against the homophily factor α obtained by the different NSUM estimators under evaluation for each party.

C Sensitivity Analysis under Homophilous Mixing

To evaluate the precision of NSUM estimators when the assumptions of random mixing are compromised due to political homophily, we conducted a sensitivity analysis using synthetic networks. Specifically, we generated stochastic block model (SBM) networks containing 250,000 nodes with a mean degree of 15. Each network was partitioned into five disjoint groups corresponding to political parties, with group prevalences reflecting the official results of the Spanish general elections in the Community of Madrid: 40.2% (PP), 27.7% (PSOE), 15.3% (Sumar), 13.9% (Vox), and 2.9% (Blank). We controlled for homophily by varying the ratio (α) between intra-group and inter-group connection probabilities. That means that a node has α times more probability of being connected with a member of its own group. Furthermore, for each network, we created 4 known-prevalence populations (10%, 5%, 2%, and 1%) to evaluate the accuracy of NSUM methods that require degree estimation, namely MoS, MLE, and PIMLE. We generated ten SBM networks for each value of α , ranging from 1 to 10, representing both weak and strong homophily effects. In this setting, the value of α equal to 10 implies that the nodes are ten times more likely to know someone from their own group, which is a significantly high level of homophily for networks with an average degree of 15.

Figure SM6 reports the estimated prevalence $\hat{\rho}$ in % as a function of α obtained by the various NSUM estimators for each party. We obtained each point on the curves by averaging the results of 2000 realizations, in which, at each trial, we extracted a sample of size $|S| = 200$ from the corresponding SBM network with a particular α . As observed, the MLE method is highly sensitive to increasing

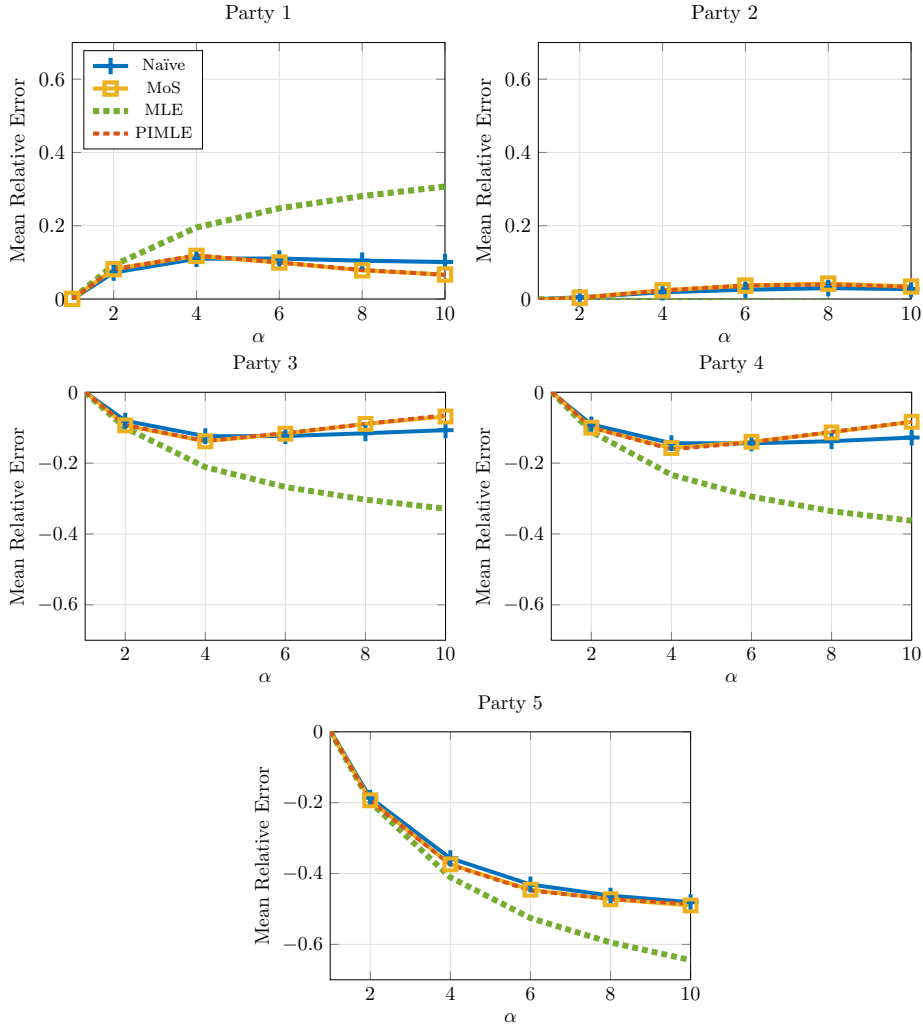


Figure SM7: Mean Relative Error versus the homophily factor α obtained by the different NSUM estimators under evaluation for each party.

homophily. For political parties with high prevalence (e.g., Party 1), the MLE increasingly overestimates prevalence as α increases, while for smaller parties (e.g., Parties 3–5), it significantly underestimates prevalence. The other estimators are affected, but the difference relative to the MLE is substantial. One possible cause is the strong random mixing assumption underlying the MLE. Concretely, MLE assumes that $y_{iu} \sim \text{Bin}(d_i, p_u)$, an assumption that is violated under homophily. In contrast, PIMLE, MoS, and Naïve only require conditions for the first moments, $E[y_{iu}] = d_i p_u$, which may still hold in some scenarios with homophily. Furthermore, the Naïve estimator is also affected, showing deviations from the true prevalence under strong homophily (large α). Although some bias is evident—particularly for medium-sized parties—the magnitude of distortion is significantly smaller than that of the MLE. Finally, the PIMLE approach closely tracks MoS across all values of α , and both outperform Naïve (which estimates the degree with the responses of the political parties), particularly with high homophily. These results suggest that correcting degrees using known populations, as done in PIMLE and MoS, partially mitigates homophily-induced bias without introducing instability. In summary, these results indicate that homophilous mixing induces systematic, directionally consistent biases whose sign and magnitude depend on group size and position on the political spectrum.

Figure SM7 displays the mean relative error as a function of the homophily factor α yielded by the various NSUM estimators for each party. In particular, the mean relative error is defined as

$$\frac{1}{|S|} \sum_{i \in S} \frac{\hat{\rho}_i - \rho}{\rho} \quad (\text{SM7})$$

where \mathcal{S} is the set of realizations. Notice that a positive error indicates overestimation, while a negative error indicates underestimation. Across all parties, the MLE method shows a rapid rise in absolute error as homophily increases, with errors ranging from $\pm 30\%$ to $\pm 60\%$ at realistic levels of homophily. This confirms that the MLE approach performs particularly poorly in politically structured networks. In contrast, Naïve, MoS, and PIMLE exhibit substantially lower relative error, even under strong homophily factors. Notably, the direction of bias is consistent: minority parties are underestimated, and majority parties are overestimated as homophily increases. Overall, the sensitivity analysis shows that the main conclusions hold across realistic levels of homophily. At the same time, these results underscore the importance of accounting for bias direction when translating vote-share estimates into downstream quantities, such as seat allocations.

D Robustness of Vote Share and Seat Allocation to Trimming and Filtering Choices

This section evaluates the sensitivity of the estimates of vote share and seat allocation to various filtering configurations. The goal is to assess whether the results reported in the main text depend on a specific, potentially ad hoc, choice of thresholds. To this end, we report results across a wide range of configurations, varying along three dimensions: (i) the trimming level applied to the voting-declaration network size (80%, 85%, 90%, and 95%), (ii) the cutoff used in the median absolute deviation (MAD) rule (3, 4, and 5), and (iii) whether the rule that removes respondents for whom more than 95% of reported alters support the same party is included or excluded. Tables SM10 and SM11 present the results for Andalusia across four estimation methods: Naïve, MoS, MLE, and PIMLE, and report both vote shares and implied seat allocations.

Across all configurations, several consistent patterns emerge. First, the estimates of vote share are remarkably stable across trimming levels and MAD cutoffs. Adjusting the trimming from 80% to 95% results in variations of generally one to two percentage points across all major parties. However, these variations do not produce significant changes in party rankings and do not systematically favor any particular method. Second, seat allocations are even more stable than vote shares. For most parties, the number of seats remains unchanged across different trimming levels, MAD cutoffs, and the implementation of the homogeneity rule. When changes occur, they are usually limited to a single seat and do not affect the overall coalition structure. Third, increasing the MAD cutoff from 3 to 5 has only a marginal effect on both vote shares and seat shares, suggesting that the estimates are not highly sensitive to extreme observations. Overall, these results indicate that the proposed approach is not significantly affected by the choice of stopping rule or filtering method. Instead, the estimates are highly robust across a wide range of commonly used trimming and outlier-handling configurations, highlighting the stability of the proposed methodology in both polarized and more heterogeneous network settings.

95% rule	MAD cutoff	Trimming	Method	Voting shares (%)					Seats			
				Vox	PP	PSOE	Sumar	Blank	Vox	PP	PSOE	Sumar
with	3	80%	Naïve	15.0	35.7	29.2	12.7	7.3	10	24	19	8
			MoS	12.9	38.2	27.9	13.7	7.2	9	26	18	8
			MLE	13.3	36.1	28.7	15.5	6.5	10	24	19	8
			PIMLE	12.4	37.4	31.3	12.8	6.2	8	25	20	8
		85%	Naïve	14.9	35.9	29.5	12.7	7.0	10	24	19	8
			MoS	12.5	38.1	29.1	13.4	6.9	9	25	19	8
			MLE	13.5	36.7	29.6	14.6	5.7	9	24	19	9
			PIMLE	12.5	37.2	31.5	12.9	6.0	8	24	21	8
		90%	Naïve	14.6	35.8	29.9	12.8	6.8	9	24	20	8
			MoS	12.8	37.4	28.9	14.0	7.0	9	25	19	8
			MLE	12.5	36.4	31.1	14.8	5.3	9	24	20	8
			PIMLE	12.3	37.0	31.5	13.4	5.8	8	24	21	8
	95%	Naïve	14.5	35.5	29.7	13.6	6.7	9	23	20	9	
		MoS	12.6	37.2	28.7	14.7	6.8	9	25	19	8	
		MLE	11.9	34.6	30.1	18.5	4.9	12	22	20	7	
		PIMLE	12.2	36.6	31.3	14.1	5.7	9	24	20	8	
	4	80%	Naïve	15.2	35.4	28.9	12.7	7.7	10	24	19	8
			MoS	13.1	37.1	28.1	13.9	7.8	9	25	19	8
			MLE	13.5	35.9	28.2	15.3	7.1	10	24	18	9
			PIMLE	12.7	37.1	30.9	12.7	6.7	8	25	20	8
		85%	Naïve	15.1	35.7	29.2	12.6	7.4	10	24	19	8
			MoS	12.7	37.5	29.0	13.5	7.2	9	25	19	8
			MLE	13.6	36.4	29.2	14.5	6.2	9	24	19	9
			PIMLE	12.8	36.9	31.1	12.8	6.4	8	24	21	8
		90%	Naïve	14.8	35.7	29.6	12.7	7.3	10	24	19	8
			MoS	13.0	37.7	28.5	13.6	7.1	9	25	19	8
			MLE	12.7	36.4	30.6	14.5	5.8	9	24	20	8
			PIMLE	12.6	36.8	31.2	13.2	6.3	8	24	21	8
	95%	Naïve	14.6	35.3	29.4	13.5	7.1	9	24	19	9	
		MoS	12.8	37.0	28.6	14.6	7.0	9	25	19	8	
		MLE	12.1	34.7	29.8	18.1	5.4	11	23	19	8	
		PIMLE	12.4	36.4	31.0	14.0	6.2	9	24	20	8	
	5	80%	Naïve	15.2	35.1	28.5	12.3	8.9	10	24	19	8
			MoS	13.1	37.1	27.6	13.4	8.8	9	25	18	9
			MLE	13.9	35.6	27.6	14.7	8.2	10	24	18	9
			PIMLE	12.7	36.6	30.6	12.3	7.8	8	25	20	8
		85%	Naïve	15.1	35.3	28.9	12.2	8.5	10	24	19	8
			MoS	13.1	36.3	28.4	13.2	9.0	9	24	19	9
			MLE	13.8	36.3	28.7	13.9	7.4	9	24	19	9
			PIMLE	12.7	36.5	30.9	12.4	7.6	8	24	21	8
		90%	Naïve	14.8	35.4	29.2	12.3	8.3	10	24	19	8
			MoS	13.0	36.8	28.1	13.6	8.5	9	25	19	8
			MLE	12.9	36.2	30.1	14.0	6.9	9	24	20	8
			PIMLE	12.5	36.4	30.9	12.8	7.4	8	24	21	8
	95%	Naïve	14.6	35.0	29.1	13.1	8.2	10	24	19	8	
		MoS	12.9	36.8	27.9	14.2	8.3	9	25	19	8	
		MLE	12.3	34.6	29.3	17.4	6.4	11	23	19	8	
		PIMLE	12.4	36.1	30.8	13.6	7.2	9	24	20	8	

Table SM10: Voting Shares and Seat Allocation for Political Groups in the July 2023 Spanish General Elections: Results from an Indirect Survey in Andalusia with Various NSUM Approaches and Filtering Configurations (with the 95% rule).

95% rule	MAD cutoff	Trimming	Method	Voting shares (%)					Seats			
				Vox	PP	PSOE	Sumar	Blank	Vox	PP	PSOE	Sumar
w/o	3	80%	Naïve	13.2	37.6	29.0	12.6	7.5	9	25	19	8
			MoS	11.3	38.6	28.9	14.0	7.2	9	26	19	7
			MLE	12.7	36.6	28.9	15.4	6.4	10	24	19	8
			PIMLE	11.1	39.6	30.6	12.6	6.1	8	26	20	7
		85%	Naïve	13.2	37.7	29.3	12.6	7.2	9	25	19	8
			MoS	11.6	39.5	28.4	13.8	6.7	9	26	19	7
			MLE	12.9	37.0	29.7	14.6	5.7	9	24	20	8
			PIMLE	11.2	39.3	30.8	12.7	5.9	8	26	20	7
		90%	Naïve	12.9	38.0	29.5	12.6	7.0	8	25	20	8
			MoS	11.0	40.3	28.2	13.9	6.6	9	27	18	7
			MLE	11.7	38.4	30.3	14.3	5.2	9	25	20	7
			PIMLE	11.0	39.6	30.6	13.0	5.7	8	26	20	7
	95%	Naïve	12.8	37.7	29.3	13.3	6.9	9	25	19	8	
		MoS	11.0	39.6	28.0	14.8	6.5	10	26	18	7	
		MLE	11.2	36.6	29.5	17.9	4.8	11	24	19	7	
		PIMLE	10.9	39.3	30.5	13.7	5.6	9	25	20	7	
	4	80%	Naïve	13.4	37.4	28.8	12.6	7.9	9	25	19	8
			MoS	11.7	39.5	27.8	13.4	7.6	9	27	18	7
			MLE	12.9	36.4	28.5	15.2	7.0	10	24	19	8
			PIMLE	11.4	39.3	30.3	12.5	6.5	8	26	20	7
		85%	Naïve	13.4	37.5	29.0	12.5	7.6	9	25	19	8
			MoS	11.9	40.3	27.6	13.1	7.1	8	27	18	8
			MLE	13.1	36.8	29.4	14.5	6.2	9	25	19	8
			PIMLE	11.5	39.0	30.5	12.6	6.3	8	26	20	7
		90%	Naïve	13.1	37.8	29.2	12.5	7.4	8	25	20	8
			MoS	11.1	39.9	27.8	14.2	6.9	9	27	18	7
			MLE	12.0	38.4	29.9	14.1	5.7	9	25	20	7
			PIMLE	11.3	39.4	30.3	12.9	6.1	8	26	20	7
	95%	Naïve	13.0	37.5	29.1	13.2	7.3	9	25	19	8	
		MoS	10.9	39.8	27.5	14.9	6.8	10	26	18	7	
		MLE	11.4	36.6	29.2	17.5	5.3	11	24	19	7	
		PIMLE	11.2	39.0	30.2	13.6	6.0	9	25	20	7	
	5	80%	Naïve	13.5	37.0	28.4	12.2	8.8	9	25	19	8
			MoS	11.6	40.1	27.0	13.0	8.4	9	27	18	7
			MLE	13.3	36.1	27.9	14.6	8.1	9	24	19	9
			PIMLE	11.4	38.8	30.1	12.2	7.6	8	26	20	7
		85%	Naïve	13.4	37.1	28.7	12.2	8.5	9	25	19	8
			MoS	12.1	39.2	27.3	12.7	8.7	8	27	18	8
			MLE	13.3	36.6	28.9	13.9	7.3	9	24	19	9
			PIMLE	11.5	38.6	30.3	12.3	7.4	8	26	20	7
		90%	Naïve	13.1	37.5	28.9	12.2	8.3	9	25	19	8
			MoS	11.5	39.3	27.8	13.2	8.2	9	26	19	7
			MLE	12.2	38.1	29.5	13.6	6.6	9	25	19	8
			PIMLE	11.3	38.9	30.1	12.6	7.1	8	26	20	7
	95%	Naïve	13.0	37.2	28.8	12.8	8.2	9	25	19	8	
		MoS	11.5	38.7	27.8	14.0	8.0	9	26	19	7	
		MLE	11.7	36.4	28.8	16.9	6.2	11	24	19	7	
		PIMLE	11.2	38.6	30.0	13.2	7.0	8	26	20	7	

Table SM11: Voting Shares and Seat Allocation for Political Groups in the July 2023 Spanish General Elections: Results from an Indirect Survey in Andalusia with Various NSUM Approaches and Filtering Configurations (without the 95% rule).